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A note on transaction costs**

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# Effects of central bank digital currencies on economic growth: A note on transaction costs

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Abstract
We study the growth effects of a central bank digital currency in a closed overlapping generations economy with an AK production technology and cash-in-advance constraints. Working generations can hold interest-bearing bank deposits as well as money balances in their portfolios. Money can be central bank digital money or physical money. Holding physical money involves transaction costs, whereas digital currency does not. Private banks intermediate transactions between workers and entrepreneurs. The results indicate that replacing physical currency with central bank digital currency lowers transaction costs, raises the balanced economic growth rate, and possibly reduces the long-term inflation and nominal interest rates.

Keywords: transaction costs, central bank digital currency (CBDC), cash-in-advance constraint, balanced growth

JEL Classification: D15, E42, E44, E58

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## 1. Introduction

Central banks such as the Federal Reserve (Fed coin), the Europe Central Bank (digital Euro), and People's Bank of China (e-yuan), are exploring the benefits and drawbacks of central bank digital currencies (CBDCs) in consideration of introducing them.[1] Bordo and Levin (2017) characterize the key features of CBDC in their seminal paper. Pfister (2019) states that the main motive for replacing paper money with CBDC could be to lower transaction costs in the sense that digital currencies save cash balances needed in transactions.[2] KPMG (2016) reports that the transaction costs of using cash and checks amount to 0.52 % of GDP in Singapore. In this paper, we examine the effect of replacing physical currency with a CBDC on long-term economic growth through changes in transaction costs in a simple endogenous growth setting in this study.

Many recent studies have examined the macroeconomic and financial stability and welfare effects brought about by the introduction of CBDCs (Agur et al., 2021; Barrdear and Kumhof, 2021; Davoodalhosseini, 2021; Fernandez-Villaverde et al., 2021). Most of them assume the coexistence of CBDC and deposits in private financial intermediates (some also have physical currency) to analyze the differences in their effects on macroeconomies. The initiation of a CBDC likely makes paper cash obsolete (Bordo and Levin, 2017). Brunnermeier and Niepelt (2019) establish a sufficient condition under which a swap of CBDC for bank deposits does not alter the general equilibrium of the economy. Andolfatto (2021) finds that the introduction of a CBDC has no detrimental effect on bank lending activity and a properly designed CBDC does not threaten financial stability. By contrast, Kim and Kwon (2022) show that, if a CBDC yields a positive interest rate, its introduction might cause financial instability of the banking system (i.e., bank panics). Keister and Sanches (2022) suggest that in designing a single *universal* digital currency, policymakers must consider tradeoffs between enhancing efficiency in exchange and crowding out bank deposits, that is, decreasing investment. Nevertheless, the literature cited above does not consider economic growth effects of the CBDC supply. Our focus in this study is the growth effects.

We consider that a CBDC is a national medium of exchange and a store of value; it is an account-based and interest-bearing claim on the central bank. It can be used in all

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[1] The People's Bank of China started demonstration experiments in several cities in China in 2020.

[2] Pfister (2019) mentions this as a motive for issuing a wholesale CBDC, which is distinguished from general-purpose CBDC in BIS (2018). It has also been highlighted that general purpose CBDCs make settlements and payments more efficient (Yanagawa and Yamaoka, 2019). In the literature, terms of general-purpose CBDC, retail CBDC and universal CBDC seem to be used interchangeably.

transactions, that is, it is a *universal* digital currency.[3] Therefore, in this study, a CBDC is regarded as a perfect substitute for physical paper money in exchange.[4] We assume that central bank determines the supply of money, both physical and digital, according to a certain price-level targeting regime in which inflation rate is kept constant over time.[5] In this study, the difference between CBDC and physical currency is that holding physical currency needs transactions costs, whereas holding CBDC does not. We focus only on this aspect of CBDC in this study. This means that individuals want to hold CBDC rather than physical money. For our analytical purposes, we assume that initially, the ratio of the CBDC in the total money holdings is constrained by the central bank to be less than one so that individuals must hold both the CBDC and physical money in their portfolios. Our research strategy is to study the effects of relaxing the constraint on the CBDC holding ratio in individuals' portfolios, and thereby analyzing the effects of introducing CBDC.

Time is discrete and runs infinitely. The lengths of the periods are normalized to unity. Each generation lives for three periods: childhood (first), youth (second), and old age (third). Individuals are looked after by their parents in the first period, earn wage income, save for retirement in the second period, and consume their savings in the third period. Retirement consumption requires that individuals hold money balances in advance. Young individuals choose portfolio assets among physical and digital money and bank deposits. Entrepreneurs invest in capital to be used in goods production in the next period. Private banks intermediate transactions between depositors and entrepreneurs. The salient features of our model setting are: (1) endogenous economic growth is driven by learning-by-doing and knowledge spillovers among workers (i.e., an AK production technology); (2) money is introduced into the model by a cash-in-advance constraint approach à la Hahn and Solow (1995)[6]; (3) transaction costs is expressed as a linear function of transacted amount of (physical) money, as introduced by Baumol (1952); and (4)

[3] We are concerned only with CBDCs and not cryptocurrencies. Schilling and Uhlig (2019) analyze the situation wherein coexistence and competition between dollars and Bitcoins (i.e., intrinsically worthless moneys). They study the behavior of the Bitcoin to dollar exchange rate and derive conditions under which Bitcoin speculation cannot occur. However, we can avoid such an issue in this study.

[4] A general-purpose, account-based (indirectly held by the central bank), non-interest-bearing CBDC is apparently considered in Japan (Bank of Japan, 2020).

[5] Keister and Sanches (2022) assume a similar central bank monetary policy.

[6] Alternatively, money has been introduced by the money-in-the-utility-function approach. Abel (1987) and Yakita (1989) consider a money-in-the-utility approach in overlapping generations settings. New monetarists such as Williamson and Wright (2010) regard the cash-in-advance approach as at best of second-order importance in modelling frictions in the exchange process.

transaction costs are real, that is, real resources are consumed.

The results are as follows. Relaxing the constraint on the CBDC ratio in individuals' money holdings reduces transaction costs of money holdings by reducing physical money holdings (i.e., improved efficiency in monetary transactions). The reduced transaction costs free the disposable resources of young workers and thereby potentially increase bank deposits. If the increased bank deposits do not lower the interest rate much, then the increased CBDC ratio induces young individuals to shift their portfolios toward bank deposits. With private bank intermediation, this results in greater investment of entrepreneurs and thereby raises the balanced growth rate. Under an AK production technology with a constant interest rate, this outcome occurs because of a constant interest rate. The reduction of transaction costs by the CBDC promotes economic growth.

The remainder of this paper is organized as follows. Section 2 introduces our model with individuals, government and central bank, private bank, and production sectors. Section 3 considers balanced growth and presents an analysis of the effects of introducing CBDC on the balanced growth rate. Finally, Section 4 concludes.

## 2. Model

We assume a closed overlapping generations economy populated by individuals who live for three periods. Each individual draws on parents in the first period (childhood), works and saves for their retirement in the second period (young), and retires in the third period (old). The population grows at a constant rate, both generation by generation and period by period. They encounter a cash-in-advance constraint: a worker needs the minimum amount of money to consume at the beginning of the retirement period. They choose portfolio assets among physical and digital money and deposits. Furthermore, transacting between consumable resources and money during the young age incurs transaction costs. Private banks intermediate between deposits of individuals and investment of entrepreneurs. Aggregate output production technology is characterized by an $AK$ technology whose engine is doing-by-doing and knowledge spillovers among workers. The government controls the money supply to keep the inflation rate constant and distributes seigniorage revenue to the working generation in each period. For expositional simplicity, we assume that there is no other government expenditure or taxes.

### 2.1 Money issue and transfer policy

The consolidated central bank issues both physical and (universal) digital currency. When the economy grows, it is necessary to issue new money to control the price level. We assume that the central bank supplies money at a rate of $\mu$ and distributes the seigniorage to working individuals as a money transfer in each period. By enforcing the same price level target for physical and digital money, the central bank should offer to convert units of physical currency one-for-one into units of digital currency and *vice versa*. Transferring physical money to workers does not imply paying interest on money holdings in this case. Nevertheless, for digital currency, it can be regarded as the interest payment. Then, the government's budget constraint is

$$\tau_t N_t = \mu(M_t / P_t) \quad \text{or} \quad \tau_t = \mu m_t, \tag{1}$$

where $\tau_t$ is a lump-sum transfer (including interest payments to CBDC) to a working individual by government, which is measured in real terms; $N_t$ is the number of individuals in their young working age; $M_t$ denotes the aggregate nominal money balance, physical and digital; and $P_t$ represents the price level in period $t$. $m_t = M_t / (P_t N_t)$ represents real money balance per worker.

## 2.2 Individuals

Individuals are homogeneous in preference and labor productivity. Each individual inelastically supplies a unit of labor in their second period of life. The budget constraints of an individual working in period $t$ in the working and retirement periods are written as

$$P_t w_t + P_t \tau_t = P_t c_t^1 + P_t s_t + (1+\mu)(M_t / N_t) + \varepsilon(\tilde{M} / N_t) + P_t z_t, \quad \text{and} \tag{2}$$

$$P_{t+1} c_{t+1}^2 = P_{t+1}(1 + r_{t+1}) s_t + (M_{t+1} / N_t), \tag{3}$$

where $c_t^1$ and $c_{t+1}^2$ are consumption when young and old, respectively; $s_t$ is real savings (i.e., bank deposits)[7]; $w_t$ is the real wage rate in period $t$; and $r_{t+1}$ is the real interest rate on real savings in period $t+1$. An individual can carry forward the received

[7] As is commonly assumed in the literature, we consider only the so-called indirect financing of entrepreneurs in this study.

money transfer to the old age period without incurring any transaction costs. By contrast, we assume here that transactions between consumable resources and physical currency require transaction costs, which is a linear function of the transacted amount of money as $\varepsilon\tilde{m}_t + z_t$, where $\tilde{m}_t = \tilde{M}_t / (P_t N_t)$ is the real physical money balance. Here, $\varepsilon > 0$ is a constant cost per transaction, and $z_t \geq 0$ is a lump-sum part of the costs. The cost might include a fee of opening a bank account. We assume that the transaction round trip of resources-(physical) cash-resources occurs only once, as is usual in overlapping generation settings.[8] Therefore, the transaction cost in (2) is the total cost of the "trip." Because of money transfers and the money supply rule of the central bank explained in the introduction, it follows that $M_{t+1} / N_t = (1+\mu) M_t / N_t$, or $(1+n)m_{t+1} = (1+\mu)m_t / (1+\pi_{t+1})$ in real terms, where $1+\pi_{t+1} = P_{t+1} / P_t$ is the (gross) inflation rate.

The budget constraints of the individual, (2) and (3), can be rewritten in real terms as follows:

$$w_t = c_t^1 + s_t + m_t + (\varepsilon\tilde{m}_t + z_t), \text{ and} \tag{4}$$

$$c_{t+1}^2 = (1+r_{t+1})s_t + \frac{1+\mu}{1+\pi_{t+1}} m_t, \tag{5}$$

where $1+n = N_t / N_{t-1}$ is a constant (gross) population growth rate.

At this stage, we consider individuals' portfolio choices by using (4) and (5). The return rate of real savings, physical money holding, and digital money holding are $1+r_{t+1}$, $(1+\mu)/[(1+\pi_{t+1})(1+\varepsilon)]$, and $(1+\mu)/(1+\pi_{t+1})$, respectively. It is readily known that

[8] Rotemberg (1984, p. 43) designates the lump-sum brokerage fee as "the individual's real cost of visiting the financial intermediary." In this study, unlike Rotemberg (1984), we assume that transaction costs are borne by individuals without paying to other agents. Kimbrough (1986) alternatively assumes the transactions time per unit of consumption depending on the ratio of money holdings to nominal consumption expenditure. By contrast, Wang and Yip (1992) instead assume a shopping-time technology to feature the transaction cost approach, where monetary transactions consume an individual's time. The last two studies consider real transaction costs in terms of time.

digital money is preferred to physical money by individuals because they are perfect substitutes. Therefore, when the nominal interest rate on deposits is lower than the interest rate on CBDC, that is, when $1+r_{t+1} < (1+\mu)/(1+\pi_{t+1})$, then individuals want to substitute real savings for digital money if possible, and *vice versa*.[9] If the central bank increases the ratio of CBDC, then individuals willingly substitute CBDC for bank deposits in their portfolios when $1+r_{t+1} < (1+\mu)/(1+\pi_{t+1})$.[10] However, recalling that the central bank constrains the ratio of individuals' CBDC holdings to be less than one. Designating the ratio as $0 \le \phi < 1$, digital money balance per worker becomes $\phi m_t$, where $\tilde{m}_t = (1-\phi)m_t$.[11]

From (4) and (5), the lifetime budget constraint of the individual is written in period-$t$ values as

$$w_t = c_t^1 + [1 - \frac{1+\mu}{(1+r_{t+1})(1+\pi_{t+1})} + \varepsilon(1-\phi)]m_t + z_t + \frac{c_{t+1}^2}{1+r_{t+1}}. \tag{6}$$

The term $(1+r_{t+1})(1+\pi_{t+1})$ is the nominal (gross) interest rate in discrete time settings, which is assumed to be greater than unity in this study.

Working individuals must hold money in advance to finance retirement consumption. The cash-in-advance constraint is assumed to be of the type assumed in Hahn and Solow (1995):

$$\theta P_{t+1} c_{t+1}^2 \le (M_{t+1}/N_t) \quad \text{or} \quad \theta c_{t+1}^2 \le (1+\mu)m_t/(1+\pi_{t+1}), \tag{7}$$

where $\theta$ is an exogenously given constant ($0 < \theta < 1$). The third-period consumption goods are purchased with the receipt from real savings as $\theta \to 0$, whereas they are purchased with money as $\theta \to 1$. Although this assumption imposes a kind of constraint on individuals' portfolios, we adopt this approach in this study for analytical simplicity.

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[9] We assume that $(1+\varepsilon) \ge (1+\mu)/[(1+r_{t+1})(1+\pi_{t+1})]$. Otherwise, an individual does not hold money. This contradicts to our assumption of cash-in-advance constraints.

[10] The logic is apparently consistent with the negative crowding-out effect of CBDC, as emphasized by Keister and Sanches (2022). By contrast, Williamson (2022) states that the disintermediation can be good for welfare in the absence of a bank monopoly.

[11] Without constraints on the CBDC holding ratio, we have $\phi = 1$. For analyzing effects of an increase in CBDC, we here rule out this case here.

We also assume that the cash-in-advance constraint is binding, as is commonly assumed in the literature; that is, $c_{t+1}^2 = (1+\mu)m_t / [\theta(1+\pi_{t+1})]$ or $m_t = c_{t+1}^2[\theta(1+\pi_{t+1})]/(1+\mu)$. Therefore, budget constraint (6) can be rewritten as

$$w_t = c_t^1 + \{1+\theta[(1+\varepsilon(1-\phi))\frac{(1+\pi_{t+1})(1+r_{t+1})}{1+\mu} - 1]\}\frac{c_{t+1}^2}{1+r_{t+1}} + z_t. \qquad (8)$$

The utility maximization problem of a working individual is to maximize their lifetime utility, subject to constraint (8). To obtain an explicit solution, we assume lifetime utility as a log-linear function of $U_t = \ln c_t^1 + \rho \ln c_{t+1}^2$, where $\rho$ is a discount factor ($0 < \rho < 1$). From the first-order conditions for utility maximization, we obtain

$$c_t^1 = \frac{1}{1+\rho}(w_t - z_t), \text{ and} \qquad (9)$$

$$c_{t+1}^2 = \frac{(1+r_{t+1})\rho}{A_{t+1}(1+\rho)}(w_t - z_t), \qquad (10)$$

where

$$A_{t+1} = 1+\theta[\frac{1+\varepsilon(1-\phi)}{1+\mu}(1+\pi_{t+1})(1+r_{t+1}) - 1]. \qquad (11)$$

We assume that condition $A_{t+1} > 0$ must hold for $c_{t+1}^2 > 0$. From (4), (7), (9), (10), and (11), with some rearrangement of terms, we obtain the optimal real savings (i.e., bank deposits) as follows:

$$s_t = \frac{\rho}{1+\rho}\frac{w_t - z_t}{1+\frac{\theta}{1-\theta}\frac{1+\varepsilon(1-\phi)}{1+\mu}(1+\pi_{t+1})(1+r_{t+1})}. \qquad (12)$$

Real savings decrease with the nominal interest rate $(1+\pi_{t+1})(1+r_{t+1})$, whereas they increase with wage income. The nominal interest rate is the opportunity cost of holding money balances. An increase in the opportunity cost also increases the price of second period consumption because it requires money in advance (see equation (8)). The transaction costs, per transaction $\varepsilon$ and lump-sum $z$, are likely to reduce the real savings because of resource costs. Nevertheless, an increase in the ratio of digital money holdings $\phi$ increases real savings and private bank deposits, *ceteris paribus*.

2.3 Production

Goods production uses labor and capital. Goods producers (i.e., entrepreneurs) borrow funds from private banks and invest in capital used in the next period. They pay the gross interest rate, which is equal to the marginal product, to the banks from which they borrow.

Aggregate production technology is assumed to be represented by a Cobb–Douglas production function:

$$Y_t = BK_t^{\alpha}(E_t N_t)^{1-\alpha} \qquad (B > 0 \text{ and } 0 < \alpha < 1), \tag{13}$$

where $Y_t$ is the aggregate output in period $t$, $K_t$ represents the aggregate capital stock in period $t$, and labor productivity is assumed to be $E_t = (K_t / N_t)/a$ ($a > 0$). This formulation is developed by Grossman and Yanagawa (1993). Under the technology, the aggregate production function can be rewritten as $Y_t = Ba^{\alpha-1}K_t$, that is, an $AK$ technology. It is well known that the first-order conditions for profit maximization are

$$1 + r_t = 1 + r = \alpha B a^{\alpha-1}, \tag{14}$$

$$w_t = (1-\alpha)Ba^{\alpha-1}k_t, \tag{15}$$

where $k_t = K_t / N_t$ is the capital–labor ratio. The interest rate remains constant over time. The wage rate increases proportionally with the capital-labor ratio. Capital fully depreciates after it is used in production in a period.

## 2.4 Private banks

Private banks intermediate funds from depositors to entrepreneurs (i.e., goods producers). For simplicity, we assume away transaction costs that are involved in these intermediations in this study.

## 2.5 Market equilibrium

The equilibrium condition in the capital market is given as $K_{t+1} = s_t N_t$, which is $(1+n)k_{t+1} = s_t$ in per worker terms. Using (12), (14), and (15), the equilibrium condition can be rewritten as

$$(1+n)k_{t+1} = \frac{\rho}{1+\rho} \frac{(1-\alpha)Ba^{\alpha-1}k_t - z_t}{1+\frac{\theta}{1-\theta}\frac{1+\varepsilon(1-\phi)}{1+\mu}(1+\pi_{t+1})(1+r_{t+1})}. \quad (16)$$

The perfectly competitive labor market equilibrium ensures full employment. The wage rate is equal to the marginal product of labor.

The money market equilibrium is represented by the condition $M_t = (1+\mu)M_{t-1}$. The left-hand side is the nominal total money demand of the young working generation, whereas the right-hand side is the total money supply of the old generation, which is inflated by newly issued money in period $t-1$. The market-clearing condition is rewritten in per worker terms as

$$(1+n)m_t = (1+\mu)m_{t-1} / (1+\pi_t). \quad (17)$$

## 2.6 Lump-sum transaction costs

We now specify the lump-sum part of the transaction costs involved in holding physical money. The lump-sum part is assumed to be proportional to the per worker output; that is,

$$z_t = b(Y_t / N_t) \quad (b \geq 0), \quad (18)$$

where $b$ is a constant. In an endogenous growth economy, it is natural to assume that lump-sum costs vary with per worker output. If the lump-sum costs are constant, then their size relative to the wage rate becomes negligible as time goes infinite. This seems to be implausible.[12] We assume that $b$ is sufficiently small, that is, $(1-\alpha) > b$.

Then, from (15) and (18), (16) can be rewritten in aggregate terms as

$$K_{t+1} = \frac{\rho}{1+\rho} \frac{[(1-\alpha)-b]Ba^{\alpha-1}}{1+\frac{\theta}{1-\theta}\frac{1+\varepsilon(1-\phi)}{1+\mu}(1+\pi_{t+1})(1+r_{t+1})} K_t. \quad (19)$$

## 3. Balanced growth

Before considering the dynamics of the system, we first consider the initial period, that is, $t = 0$. The initial levels of $K_0$, $N_{-1}$, and $M_0$ are given. Money balances are

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[12] All transaction costs would disappear totally with full digitalization, although we rule out this possibility in this study.

shared equally by the old generation $N_{-1}$. From (3), we have $c_0^2 = (1+r)(K_0 / N_{-1}) + (M_0 / N_{-1}P_0)$, where the price level $P_0$ has been determined in period $t-1$. The price level in period $t=1$ or, equivalently, the inflation rate $\pi_1 = (P_1 / P_0) - 1$, is determined in period $t=0$ so that consumption plan $c_2^2$ satisfies the cash-in-advance constraint with equality.[13]

We define a balanced growth path as a path on which the aggregate real endogenous variables grow at the same rate, $\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{M_{t+1}/P_{t+1}}{M_t/P_t} (= \frac{(1+n)m_{t+1}}{m_t}) \equiv 1+\gamma$, where $\gamma$ designates the balanced growth rate.[14] The dynamics of the system are represented by the difference equation (19), which has two endogenous variables, $K_t$ and $\pi_t$. Completing the dynamic analysis requires one more equation governing the movement in $\pi_t$.[15] Nevertheless, it is difficult to solve the dynamics explicitly in a dynamic system of the two equations. In this study, we are concerned with the effect of the changes in the transaction costs on the long-term growth rate. Therefore, we assume the existence and stability of such a balanced growth path. If the central bank keeps issuing growth money on the balanced growth path, that is, controlling $\mu$ to satisfy condition $1+\mu = 1+\gamma$, then we have $\pi = 0$ on the balanced growth path from (17). This is considered as the price-level target discussed in the introduction. In this case, the targeted inflation rate is zero. Although this assumption seems to be restrictive, it is useful for isolating the effects of transaction costs on balanced growth, thereby neutralizing the effects of inflation. Recall that our purpose is to present an analysis of the effects of replacement of physical currency with digital currency on balanced growth rates.

Under the monetary policy of $1+\mu = 1+\gamma$, that is, when $\pi = 0$, the economy achieves a balanced growth rate.[16] The balanced growth rate is obtained from (19) as

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[13] The resource constraint is explained in Appendix A.

[14] We assume that $\gamma > n$ as is commonly assumed in the literature. Otherwise, per worker income approaches zero as time approaches infinity.

[15] The other difference equation of $K_t$ and $\pi_t$ is explained in Appendix B. If the system is not stable, then examining the comparative static effects of parameter changes makes no sense.

[16] The case in which the money supply rate (and the CBDC interest rate) is kept constant

$$1+\gamma=\frac{\rho}{1+\rho}\frac{[(1-\alpha)-b]Ba^{\alpha-1}}{1+\frac{\theta}{1-\theta}\frac{1+\varepsilon(1-\phi)}{1+\mu}(1+r)}, \tag{20}$$

provided the balanced growth path is stable. From (20) we obtain the following comparative static results:[17]

$$\frac{d\gamma}{d\phi}>0,\ \frac{d\gamma}{d\varepsilon}<0 \text{ and } \frac{d\gamma}{db}<0. \tag{21}$$

Summarizing the above arguments, we obtain the following propositions.

**Proposition 1** *Assume a stable balanced growth path on which growth money is supplied. Then, an increase in the ratio of the CBDC ($\phi$) raises the growth rate.*

**Proposition 2** *Assume a stable balanced growth path on which growth money is supplied. Then, a decrease in the cost per unit of transactions ($\varepsilon$) and a decrease in lump-sum transaction costs ($b$) raise the growth rate.*

We now discuss the underlying intuition behind the propositions. The increased CBDC ratio induces individuals to hold more digital money in their portfolios, instead of physical money, which incurs transaction costs when held. In other words, individuals can reduce transaction costs of holding total money by increasing the ratio of digital money holdings in their portfolios in response to the policy change. The decreased transaction costs raise their disposable resources by freeing real resources, *ceteris paribus*. The increased disposable resources induce young individuals to accelerate their asset accumulation and hence, in this case, bank deposits. Thus, capital accumulation is accelerated through bank intermediation. Because capital accumulation is the engine of this model, the decreased transaction costs boost economic growth. It is noteworthy here that an increased CBDC does not crowd out bank deposits through changes in the interest rate in this model (see Subsection 2.2). In our model with AK production technology, the interest rate is independent of the capital–labor ratio and remains constant over time.

A decrease in the lump-sum cost factor also lowers the cost burden and frees disposable resources. Therefore, lump-sum cost reduction boosts economic growth. This result is not

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at $1+\mu\neq 1+\gamma$ is provided in Appendix C.

[17] It is noteworthy that $1+\mu=1+\gamma$ on the balanced growth paths with growth-money supply policies. Therefore, (20) is an implicit equation of the balanced growth rate. Under the money supply policy, (20) becomes $1+\gamma=\rho(1-\alpha)Ba^{\alpha-1}/(1+\rho)-\theta[1+\varepsilon(1-\phi)](1+r)/(1+\theta)$.

contrary to the arguments of Rotemberg (1984), who assumes a neoclassical infinitely lived representative agent model. However, in this study, the costs are not paid to financial brokers; in other words, they are real resource costs. If CBDCs lower monetary transaction costs, then the introduction or replacement of CBDCs promotes economic growth.

Under Propositions 1 and 2, the growth money supply policy maintains the long-term inflation rate at zero. Therefore, the money supply policy has no substitution effects between physical capital and real balances; that is, there is no Tobin effect (Tobin, 1956).[18]

## 4. Concluding remarks

Our results show that if a balanced growth path exists, increases in the ratio of CBDC raise economic growth. Individuals' portfolio adjustments reduce transaction costs of holding (total) money. This is true regardless of the government's money supply rule. In other words, if CBDCs reduce transaction costs, the replacement of physical money with a CBDC exerts positive effects on economic growth by freeing the disposable resources to investment if the real interest rate does not change much. The replacement with CBDC potentially lowers the inflation rate and hence the nominal interest rate.

Because of a simple model, our study has drawbacks. A possible analytical extension would be to consider a different endogenous growth setting, such as that with an R&D investment engine. When the interest rate is endogenized, the interest rate on the CBDC might affect the real interest rate (Keister and Sanches, 2022). We have not addressed on the importance of privacy in transactions, which is for example emphasized in Williamson (2022). Another extension is to consider the effects of other policies in addition to money supply and lump-sum transfer policies. The optimum can be attained with multiple policy measures in overlapping generations settings (Crettez et al., 2002; Gahvari, 2007).

## Appendix A: Walras' law and the resource constraint

We have $Y_t = (1+r)K_t + w_t N_t$ from (13), (14), and (15); $w_t N_t + \tau_t N_t = c_t^1 N_t + s_t N_t + [(1+\mu)m_t + \varepsilon \tilde{m}_t] N_t + z_t N_t$ from (4);

[18] A constant real interest rate implies that the nominal interest rate also remains constant in the long term.

$(1+r)K_t + m_t N_t = c_t^2 N_{t-1}$ from (5) and $(1+n)N_{t-1} = N_t$ ; $\tau_t N_t = \mu m_t N_t$ from (1); and $s_t N_t = K_{t+1}$ from the capital market-clearing condition. From these equations, we obtain the resource constraint:

$$Y_t = c_t^1 N_t + (1-\theta)c_t^2 N_{t-1} + K_{t+1} + (m_t + \varepsilon \tilde{m}_t + z_t)N_t \text{, or}$$

$$Y_t = c_t^1 N_t + c_t^2 N_{t-1} + K_{t+1} + (\varepsilon \tilde{m}_t + z_t)N_t \, , \tag{A1}$$

where $\theta c_t^2 N_{t-1} = m_t N_t$ is from the cash-in-advance constraint. The left-hand side of (A1) is the aggregate output, which can be consumable and investable. The right-hand side is the sum of the goods consumption of the two generations, physical capital accumulation, and real resource costs of money transactions. The resource costs of money transactions are borne by individuals, such as the sunk costs of monetary transactions, without payment to any party in this study.

Appendix B: Balanced growth path

From budget constraint (3) and the cash-in-advance constraint, we obtain

$$c_{t+1}^2 = (1+r)s_t / (1-\theta) \, . \tag{A2}$$

Using the cash-in-advance constraint, we have

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}/(c_{t+1}^2 N_t)}{M_t/(c_t^2 N_{t-1})} = \frac{M_{t+1}}{M_t}\frac{s_{t-1}}{s_t}\frac{1}{1+n} = \frac{1+\mu}{1+n}\frac{s_{t-1}}{s_t} \, . \tag{A3}$$

From (12) and (A3), we obtain

$$1+\pi_{t+1} = \frac{1+\mu}{1+n}\frac{[1+\dfrac{\theta}{1-\theta}\dfrac{1+\varepsilon(1-\phi)}{1+\mu}(1+\pi_t)(1+r)][(1-\alpha)-b]Ba^{\alpha-1}k_{t-1}}{[1+\dfrac{\theta}{1-\theta}\dfrac{1+\varepsilon(1-\phi)}{1+\mu}(1+\pi_{t+1})(1+r)][(1-\alpha)-b]Ba^{\alpha-1}k_t} \, , \tag{A4}$$

where we use (18) and $Y_t = Ba^{\alpha-1}K_t$ . In period $t$ , the variables $\pi_t$ , $k_{t-1}(=K_{t-1}/N_{t-1})$, and $k_t(=K_t/N_t)$ have already been determined. Therefore, (A4) determines $\pi_{t+1}$ for these predetermined variables. (A4) may yield multiple solutions.

The dynamic stability must be determined using (16) and (A4). However, it is also difficult to study the stability and uniqueness of dynamic systems.

It is noteworthy that (A3) shows $P_{t+1}/P_t = 1$ on the balanced growth path if the growth money is provided (i.e., $1+\mu = 1+\gamma$), because $(c_{t+1}^2 N_t)/(c_t^2 N_{t-1}) = 1+\gamma$. When $\pi_t = 0$, the dynamics of the system are given by (19).

$$K_{t+1} = \frac{\rho}{1+\rho} \frac{1}{1+\dfrac{\theta}{1-\theta}\dfrac{1+\varepsilon(1-\phi)}{1+\mu}(1+r)} [(1-\alpha)-b]Ba^{\alpha-1}K_t\,. \tag{A5}$$

Because capital stock grows on a balanced growth path, we obtain a balanced growth rate as presented by (20).

Appendix C: Constant money supply rate

In Appendix C, we assume that the money supply rate is constant. If the money supply policy is not for growth money supply (i.e., if $1+\mu \neq 1+\gamma$), then (A4) provides the dynamics of the inflation rate. On the balanced growth path on which $1+\gamma[=(1+n)m_{t+1}/m_t] = (1+\mu)/(1+\pi)$ for a given constant money supply rate $\mu$, we have $1+\pi = (1+\mu)/(1+\gamma)$. Therefore, it follows that $\pi > (<) 0$ when $\mu > (<) \gamma$. When the money supply rate is higher (lower) than the balanced growth rate, then the inflation rate is positive (negative).

It is noteworthy that the balanced growth and inflation rates are determined simultaneously by the two dynamic equations, (16) and (A4). Although it is also difficult to examine stability even if we assume the existence of balanced growth paths, we obtain the comparative static results as

$$\frac{d\gamma}{d\mu} = 0 \quad \text{and} \quad \frac{d\pi}{d\mu} > 0\,. \tag{A6}$$

$$\frac{d\gamma}{d\phi} > 0 \quad \text{and} \quad \frac{d\pi}{d\phi} < 0\,. \tag{A7}$$

Therefore, under a constant ratio of CBDC with a constant money supply rate policy, an increase in the money supply rate (i.e., CBDC interest rate) does not affect the balanced growth rate and increases the inflation rate. An increase in the CBDC ratio increases the balance growth rate and reduces the inflation rate. The growth effects of the transaction costs $\varepsilon$ and $b$ are qualitatively the same as those in (21). The superneutrality result in

(A6) comes from the assumed monetary policy in this case, in which the increased money supply rate does not affect the discounted return rate to real money holding (see Crettez et al., 1999).

Declaration of Conflicting Interests

The authors declare no potential conflict of interest with respect to the study, authorship, or publication of this article.

Data Availability Statement

Data sharing is inapplicable to this paper because no new data were analyzed.

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