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Time inconsistency of a Central Bank in a Monetary Search Model with Agent's Time inconsistency

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Time Inconsistency of a Central Bank in a Monetary Search Model with Agent's Time Inconsistency*

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Abstract

I incorporate the time-inconsistent preference for hyperbolic discounting into a monetary search model following Lagos and Wright (2005) and use it to analyze two economies. One economy consists of sophisticated agents who understand their time inconsistency, whereas the other consists of naïve agents who do not understand their time inconsistency. I extend previous analyses of this topic by considering two monetary policy rules: *inflation targeting* under which the target variable is the inflation rate and *nominal growth rate targeting* under which the target variable is the growth rate of gross domestic product. Through this analysis, I show that inflation targeting is a time-inconsistent monetary policy rule in the economy consisting of the naïve agents even if there is no uncertainty.

Keywords: Hyperbolic discounting; Monetary search model; Monetary policy; Inflation targeting

JEL *classification*: E52; E70

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1 Introduction

Central banks suffer time inconsistency because of the trade-off between inflation and output. If the central bank has the discretion to decide on monetary policy, it has an incentive to raise inflation from its pre-planned rate to increase output. Since high inflation hurts agents' welfare, many studies suggest policy rules to avoid it (e.g., inflation targeting under which the target variable is the inflation rate). However, previous studies analyze only the central bank's time inconsistency and not that of the agent. If agents have time inconsistency, they may change their planned behavior, meaning that the central bank may also need to change its predetermined monetary policy. Therefore, in this study, I investigate monetary policy rules in the economy with agents' time inconsistency.

In the presented theoretical analysis, I incorporate hyperbolic discounting, a well-known discounting method that causes time inconsistency, into the monetary search model provided by Lagos and Wright (2005), called the LW model hereafter. I consider two types of economies. One consists of sophisticated agents who understand their time inconsistency (called the sophisticated agent economy hereafter) and the other consists of naïve agents who do not understand their time inconsistency (called the naïve agent economy hereafter). These economies do not face macroeconomic shocks.

This study's main contribution is to extend previous analyses of this topic by considering two monetary policy rules: *inflation targeting* under which the target variable is the inflation rate and *nominal growth rate targeting* under which the target variable is the growth rate of gross domestic product (GDP). I show that inflation targeting is a time-inconsistent monetary policy rule in the naïve agent economy because the target of the inflation rate is not achieved by the central bank that maximizes agents' welfare. Because of this time inconsistency of the central bank, this result shows that its monetary policy may not be trusted by agents and that inflation targeting may be unable to fulfill its role, which stabilizes agents' expectations. This result is not obtained in the sophisticated agent economy and when the central bank adopts nominal growth rate targeting.

I review related literature to bring a perspective. The standard models discuss monetary policy rules by defining or deriving a loss function. This increases if the inflation rate or the output gap increases. Since there are several pieces of literature using the loss function, I study a recent paper. Billi (2017) discusses the optimality of GDP targeting when the nominal interest rate hits the zero lower bound by using the New Keynesian model. Another literature that discusses the central bank's targeting rules is reviewed by Walsh (2010).

Some studies examine policy rules or the stabilization of monetary policy by using the monetary search model. Berentsen and Waller (2011) show that price-level targeting is the optimal policy rule for the short term. If the current price deviates from the long-term price path due to shocks, the central bank controls the amount of money to be injected to return the price to the target level. Berentsen and Waller (2015) discuss that when the probability of a successful transaction is determined by the number of agents entering the market, the optimal stabilization monetary policy can be derived. Boel and Waller (2019) discuss the optimal stabilization policy at the zero lower bound when agents face idiosyncratic preference shocks. However, while these studies focus on the buyer's liquidity constraint, which is relaxed when the central bank injects money, my study focuses on the agent's forecast error due to their time-inconsistent preference.

Some studies assess monetary policy rules in the economy in which agents or firms cannot correctly predict the future. Ball, Mankiw, and Reis (2005) develop a model in which none of the firms can immediately obtain the required information to set their prices, as in Mankiw and Reis (2002). They indicate that price-level targeting is the more optimal policy than inflation targeting. Agliari, Massaro, Pecora, and Spelta (2017) study a New Keynesian model in which there are costs to obtain information on economic states, as in Sims (2003). They also prove that the achievement of the inflation rate target depends on the costs. Angeletos and La'O (2020) study a model in which the signals of the economic state differ by firm type, as in Morris and Shin (1998). They show that nominal GDP targeting is more desirable than price-level targeting. In addition, Grauwe (2011), Ho, Lin, and Yeh (2021), and Molnar (2020) incorporate heuristics into the New Keynesian model. However, the cause of the agent's forecast error in these models differs from those in my model.

The following studies examine monetary policy using a behavioral economics model. Graham and Snower (2008, 2013) and Maeda (2018, 2023) study the case of hyperbolic discounting. Hori, Futagami, and Morimoto (2021) and Futagami and Maeda (2023) study non-unitary discounting in which discount rates differ between goods. Hiraguchi (2018) studies a model in which agents have the temptation to spend all their money. However, these studies do not discuss the monetary policy rules presented in my study.

The rest of the paper is structured as follows. Section 2 explains the setting of the model. Section 3 solves the agent's optimization problem. Section 4 derives the equilibrium and discusses the optimality of inflation targeting. Section 5 concludes.

2 The model

There is a continuum of agents with a unit measure. The agents live infinitely. Time is discrete and goes from $t = 0$ to ∞. Each period has two subperiods: "day" and "night." Each subperiod has a goods market and the goods traded in them cannot be carried over from one subperiod to the next.

Before the agents enter the day subperiod, they are divided into "sellers" and "buyers." The agents become sellers with the probability $n \in (0, 1)$ and buyers with the probability $1 - n$. During the day, the sellers produce and sell goods and the buyers buy and consume them. To focus on the agent's time inconsistency, I simply assume that the goods produced by each seller are the same, following Berentsen, Camera, and Waller (2007). Then, the market is perfectly competitive[1]. This assumption seems to be the same as a centralized market. However, I call this market a decentralized market (DM) following the original

[1] Random matching and Nash bargaining can be incorporated into the decentralized market to determine the quantity and price of goods as in the original LW model. However, the difference between the original setting and setting of the perfectly competitive, decentralized market in my model with time inconsistency is qualitatively the same as the difference in the model without time inconsistency because the Nash bargaining process is not relevant to intertemporal decision-making.

setting of the LW model. I assume that there are no methods of trade record-keeping. Therefore, the buyers have to use money to buy the goods.

At night, all the agents simultaneously decide the consumption, labor supply, and money holdings to be carried over to the next period. These markets are the same as in the standard model (i.e., a centralized, perfectly competitive market). I call the goods market at night a centralized market (CM). In this market, a unit of goods is produced by a unit of labor. Since I assume that the labor market is perfectly competitive and the good traded in the CM (i.e., the CM good) is a numeraire good, the real wage is equal to one.

Money is divisible and storable, but intrinsically useless. M_t is the amount of money issued before period t. The central bank issues and transfers money to the agents equally at night via lump-sum transfers. I define T_t as the transfer of money to agents. Then, $T_t = M_{t+1} - M_t$.

Next, I explain the agent's preference. The agent obtains utility from consumption and disutility from production and labor supply in each subperiod. I assume that the utility function is additively separable and quasilinear:

$$\mathcal{U}(q_t^b, q_t^s, x_t, h_t) = u(q_t^b) - c(q_t^s) + U(x_t) - h_t, \tag{1}$$

where $u(q)$ is the utility from consuming q units of goods and $c(q)$ is the cost of producing q units of goods during the day. $U(x)$ is the utility from consuming x units of goods and h is the disutility of supplying h units of labor during the night. The functions u, c, and U are twice continuously differentiable and satisfy $u(0) = c(0) = 0,\ u' > 0, c' > 0,\ U' > 0,\ u'' < 0\ c'' \geq 0,$ $U'' < 0,\ u'(0) = U'(0) = \infty$, and $u'(\infty) = U'(\infty) = 0$. h is positive and has the upper bound $\bar{h}$. As mentioned in the Introduction, future utility is hyperbolically discounted. Therefore, lifetime utility in period t is denoted by

$$Z_t = u(q_t^b) - c(q_t^s) + U(x_t) - h_t + \beta \sum_{i=1}^{\infty} \delta^i \left[u(q_{t+i}^b) - c(q_{t+i}^s) + U(x_{t+i}) - h_{t+i} \right], \tag{2}$$

where $\delta \in (0, 1)$ is the long-term discount factor. Since I assume that $\beta \in (0, 1]$, the agents evaluate current utility rather than future utility. In other words, β is a parameter denoting present bias.

Behavioral economics studies consider two types of agents. First, sophisticated agents understand that their preference changes over time. In this model, they know that their future selves will also have the present bias parameter β and resolve the optimization problem. In other words, they know that their future selves will maximize (2). Second, naïve agents do not understand that their preference changes over time. In this model, they understand that their future selves will not have present bias and their current decision is not changed by their future selves. In other words, they understand that their future selves will maximize $\sum_{i=1}^{\infty} \delta^i \mathcal{U}(q^b_{t+i}, q^s_{t+i}, x_{t+i}, h_{t+i})$. I summarize the agents' types as follows.

Definition 1. *In this model, sophisticated and naïve agents predict their future behavior as follows*

1. *Sophisticated agents predict that their future selves will maximize* (2).

2. *Naïve agents predict that their future selves will maximize* $\sum_{i=1}^{\infty} \delta^j \mathcal{U}(q^b_{t+i}, q^s_{t+i}, x_{t+i}, h_{t+i})$.

In this study, I consider two cases: all the agents are sophisticated and all the agents are naïve.

3 The agent's optimization problem

3.1 The current problem

Before I discuss the agent's optimization problem, note that I omit the variables' subscript, t, and add $+1$ to the variables in the next period.

I start by solving the seller's problem by using the Bellman equation. An individual experiences disutility $c(q^s)$ by producing the goods during the day (i.e., DM goods) and experiences net utility $U(x) - h$ during the night. I define the seller's problem as follows:

$$V_0^s(m) = \max_{q^s, x, h, m_{+1}} [-c(q^s) + U(x) - h + \beta\delta V(m_{+1})], \tag{3}$$

where m denotes money holdings. $V(m)$ denotes the expected value obtained from future consumption and production (i.e., the sum of the expected discounted future utility). The definition of $V(m)$ is different for sophisticated and naïve agents, as explained after this subsection. The agent obtains nominal income pq^s, where p is the nominal price of the DM goods from production during the day. When ϕ denotes the real price of money (the inverse of the price of the goods in the CM), real income is given by ϕpq^s. As mentioned before, the real wage at night is one. Thus, the seller's budget constraint is denoted by

$$h = x + \phi m_{+1} - \phi pq^s - \phi(m + T). \tag{4}$$

Substituting (4) into (3), I obtain

$$V_0^s(m) = \max_{q^s}[\phi pq^s - c(q^s)] + \max_{x}[U(x) - x] + \phi(m + T) + \max_{m_{+1}}[-\phi m_{+1} + \beta\delta V(m_{+1})]. \tag{5}$$

Since the DM is perfectly competitive, the seller behaves with price ϕp as given. Then, the first-order condition of q^s is

$$\phi p = c'(q^s). \tag{6}$$

From the second term of (5), the first-order condition of x is given by

$$U'(x) = 1. \tag{7}$$

Let x^* denote the value of satisfying (7). Moreover, I assume that $U(x^*) > x^*$ is satisfied for

x^* to have a positive value. From the last term of (5), I find that m_{+1} does not depend on m or on whether the individual is a seller or a buyer during the day. The first-order condition of m_{+1} is presented in the next subsection.

Next, I solve the buyer's problem. Since the buyers are anonymous, they have to hold money before they consume it. Therefore, the buyer faces the following constraint:

$$pq^b \leq m. \tag{8}$$

The buyer's budget constraint is given by

$$h = x + \phi m_{+1} + \phi p q^b - \phi(m + T). \tag{9}$$

As for the seller's problem, the value function of the buyer is calculated by

$$V_0^b(m) = \max_{pq^b \leq m} [u(q^b) - \phi p q^b] + \max_x [U(x) - x] + \phi(m + T) + \max_{m_{+1}} [-\phi m_{+1} + \beta\delta V(m_{+1})]. \tag{10}$$

From (10), the buyer's optimal conditions of x and m_{+1} are the same as those of the seller. If (8) binds, $q^b = m/p$. If it does not bind, $q^b = q^*$, which satisfies $u'(q^*) = \phi p$. Hereafter, I consider the case in which (8) binds[2].

3.2 The sophisticated agent's expectation and money holdings

I need to know the future value $V(m)$ to determine money holdings. $V(m)$ is the sum of the expected discounted future utility as follows:

$$V(m) = E \sum_{i=0}^{\infty} \delta^i \left[u(q_{t+i}^b) - c(q_{t+i}^s) + U(x_{t+i}) - h_{t+i} \right], \tag{11}$$

[2] I do not focus on the case in which (8) does not bind because the agent's welfare in this case is no higher than the case in which (8) binds.

where E is an expectation operator. Since the probabilities of the individuals becoming sellers and buyers are n and $1-n$, respectively, I can rewrite (11) as follows:

$$V(m) = (1-n)u(q^b_{so}) - nc(q^s_{so}) + [(1-n)U(x^b_{so}) + nU(x^s_{so})] - [(1-n)h^b_{so} + nh^s_{so}] + \delta V(m_{so,+1}), \tag{12}$$

where the superscripts b and s, respectively, denote the values when the individual will become a buyer and a seller in the future, and the subscript so denotes a sophisticated agent's expected value. From definition 1, individuals predict that their future selves maximize (2); that is, the predicted future objective function is the same as the current one. This means that their future selves' behavior is the same as their behavior in the present. Since q^s, x and m_{+1} do not depend on m (see Section 3.1), I can rewrite the last terms of (5) and (10) as follows:

$$\max_{m_{+1}}[-\phi m_{+1} + \beta\delta\{(1-n)v(m_{+1}) + \phi_{+1}m_{+1}\}], \tag{13}$$

where $v(m) \equiv u(q^b) - \phi pq^b$, and I omit the variables that do not depend on m_{+1}. The optimal condition of money holdings is denoted by

$$\frac{\phi}{\phi_{+1}} = \beta\delta\left[(1-n)\frac{u'(q^b_{+1})}{\phi_{+1}p_{+1}} + n\right]. \tag{14}$$

Using the real price of the DM goods obtained by (6), I can rewrite (14) as follows:

$$\frac{\phi}{\phi_{+1}} = \beta\delta\left[(1-n)\frac{u'(q^b_{+1})}{c'(q^s_{+1})} + n\right]. \tag{15}$$

The left-hand side of (15) is the gross inflation rate. Therefore, (15) expresses the relationship between the inflation rate and output. As explained in Section 4, the market-clearing condition of the DM is given by $q^b = \frac{1-n}{n}q^s$. Moreover, I assume $u'' < 0$ and $c'' \geq 0$. Therefore, output decreases if the inflation rate increases.

3.3 The naïve agent's expectation and money holdings

From definition 1, the naïve agents predict they would maximize $\sum_{i=0}^{\infty} \delta^j \mathcal{U}(q^b_{t+j+i}, q^s_{t+j+i}, x_{t+j}, h_{t+j+i})$ in the future. This is the same as maximizing $V(m)$. Therefore, I can rewrite (11) as follows:

$$V(m) = \max_{q^b_{na}, q^s_{na}, x_{na}, h_{na}} [E \sum_{i=0}^{\infty} \delta^i \left[u(q^b_{na,t+i}) - c(q^s_{na,t+i}) + U(x_{na,t+i}) - h_{na,t+i} \right], \tag{16}$$

where the variables added to the subscript na denote the naïve agent's expected value. As in Subsection 3.1, I first solve the future seller's problem. The future seller's value is denoted by

$$V^s(m_{na}) = \max_{q^s_{na}, x_{na}, h_{na}, m_{na,+1}} [-c(q^s_{na}) + U(x_{na}) - h_{na} + \delta V(m_{na,+1})]. \tag{17}$$

Substituting the seller's budget constraint, (4), into (17), I obtain

$$\begin{aligned} V^s(m_{na}) = &\max_{q^s_{na}} [\phi_{na} p_{na} q^s_{na} - c(q^s_{na})] + \max_{x_{na}} [U(x_{na}) - x_{na}] + \phi_{na}(m_{na} + T_{na}) \\ &+ \max_{m_{na,+1}} [-\phi_{na} m_{na,,+1} + \delta V(m_{na,+1})]. \end{aligned} \tag{18}$$

From this equation, I obtain the following optimal conditions:

$$\phi_{na} p_{na} = c'(q^s_{na}), \tag{19}$$

$$U'(x_{na}) = 1. \tag{20}$$

Since (20) is the same as (7), $x_{na} = x^*$.

The future buyer's problem is denoted by

$$V^b_{na}(m_{na}) = \max_{p_{na}q^b_{na} \le m_{na}} [u(q^b_{na}) - \phi_{na}p_{na}q^b_{na}] + \max_{x_{na}}[U(x_{na}) - x_{na}] + \phi(m_{na} + T_{na})$$
$$+ \max_{m_{na,+1}} [-\phi m_{na,+1} + \delta V(m_{na,+1})], \tag{21}$$
$$s.t. \quad (8) \; and \; (9).$$

Therefore, since I assume that (8) binds, $q^b_{na} = m_{na}/p_{na}$. The optimal condition of x_{na} is denoted by (20).

Next, I find the expected money holdings, $m_{na,+1}$. I can rewrite (16) as follows:

$$V(m_{na}) = (1-n)u(q^b_{na}) - nc(q^s_{na}) + [(1-n)U(x^b_{na}) + nU(x^s_{na})] - [(1-n)h^b_{na} + nh^s_{na}] + \delta V(m_{na,+1}). \tag{22}$$

As in the current problem, I can rewrite the last terms of (18) and (21) as follows:

$$\max_{m_{na,+1}} [-\phi_{na}m_{na,+1} + \delta\{(1-n)v(m_{na,+1}) + \phi_{na,+1}m_{na,+1}\}]. \tag{23}$$

If $q^b_{na} = m_{na}/p_{na}$, the optimal condition of the expected money holdings is denoted by

$$\frac{\phi_{na}}{\phi_{na,+1}} = \delta\left[(1-n)\frac{u'(q^b_{na,+1})}{\phi_{na,+1}p_{na,+1}} + n\right]. \tag{24}$$

I obtain the expected values of the naïve agents from conditions (19), (20), and (24) and use them to find current money holdings. The current problem is the same for the sophisticated and naïve agents. Therefore, I solve the problem corresponding to (13), which is denoted by

$$\max_{m_{+1}}[-\phi m_{+1} + \beta\delta\{(1-n)v_{na}(m_{+1}) + \phi_{na,+1}m_{+1}\}], \tag{25}$$

where $v_{na}(m_{+1}) = u(q^b_{na}) - \phi_{na,+1}p_{na,+1}q^b_{na,+1}$ and $q^b_{na} = m_{+1}/p_{na,+1}$. I do not have to replace

$m_{na,+1}$ with m_{+1} because the current agents can decide current nominal money holdings. Solving (25), I obtain

$$\frac{\phi}{\phi_{na,+1}} = \beta\delta\left[(1-n)\frac{u'(q^b_{na,+1})}{\phi_{na,+1}p_{na,+1}} + n\right]. \tag{26}$$

To determine the agents' expectations, I need their price expectations. The prices are determined in each market. Therefore, I also need an agent's expectation of the other agents' behavior. I make the following assumption by referring to Gabrieli and Ghosal (2013), Ojima (2017), and Futagami and Maeda (2023).

Assumption 1. *A naïve agent predicts that the other agents also behave as naïve agents.*

From the above assumption and the agents' utility functions being the same, the DM goods' price is given by (19). Substituting this into (24) and (26), I obtain

$$\frac{\phi_{na}}{\phi_{na,+1}} = \delta\left[(1-n)\frac{u'(q^b_{na,+1})}{c'(q^s_{na,+1})} + n\right], \tag{27}$$

$$\frac{\phi}{\phi_{na,+1}} = \beta\delta\left[(1-n)\frac{u'(q^b_{na,+1})}{c'(q^s_{na,+1})} + n\right]. \tag{28}$$

These equations express the negative relationship between the expected inflation rate and expected output, as in Subsection 3.2.

4 Equilibrium and policy

4.1 Market-clearing conditions

Since I assume that the probabilities of becoming a seller and a buyer are n and $1-n$, respectively, and there is a continuum of agents with a unit measure, n sellers and $1-n$

buyers exist. Therefore, the market-clearing condition of the DM is denoted by

$$(1-n)q^b = nq^s. \tag{29}$$

Hereafter, I define q as the buyer's consumption. Therefore, $q^s = [(1-n)q]/n$. Since the productivity of labor is one and the consumption of the CM goods is denoted by (6), the market-clearing condition of the CM is denoted by

$$(1-n)x^b + nx^s = (1-n)h^b + nh^s, \tag{30}$$

where x^j $j \in b, s$ is the consumption of the CM good by the buyer and sellers. Since the agents' money holdings must be equal to the money supply, I obtain

$$m = M. \tag{31}$$

4.2 Policy rules

In this study, I consider two monetary policy rules: inflation targeting and nominal growth rate targeting. First, following previous studies on the LW model, I define $\phi/\phi_{+1} - 1$ as the inflation rate in this model because ϕ is the reciprocal number of the price of the numeraire good. Therefore, setting the inflation rate as in the following equation is termed inflation targeting herein:

$$\frac{\phi}{\phi_{+1}} = \Pi, \tag{32}$$

where Π is constant and larger than zero.

Second, since the agents in this economy consume the DM and CM goods and there are no investment goods, nominal GDP is given by

$$GDP \equiv pq + \frac{1}{\phi}x^*. \tag{33}$$

CM goods consumption is given by x^* because all the agents consume x^* units of CM goods from (7). As nominal growth rate targeting aims to keep the growth rate of (33) constant, the policy rule is given by

$$\frac{p_{+1}q_{+1} + \frac{1}{\phi_{+1}}x^*}{pq + \frac{1}{\phi}x^*} = g, \tag{34}$$

where g is constant and larger than zero.

4.3 The equilibrium path in the sophisticated agent economy

To find the equilibrium path in the sophisticated agent economy, I define the equilibria as follows.

Definition 2. *Given a monetary policy rule, the equilibria in the sophisticated agent economy consist of the quantities* $\{q, x, m_{+1}, h\}$ *and prices* $\{\phi, p\}$*, which satisfy the following conditions.*

1. *The optimal conditions of the agent's behavior are given by* (6), (7)*, and* (15)*.*
2. *The agents' budget constraints are given by* (4) *and* (9)*.*
3. *The market-clearing conditions are given by* (29)–(31)*.*
4. *Constraint* (8) *binds.*

First, I obtain the equilibria when the monetary policy rules are inflation targeting. Since the policy rule of inflation targeting is given by (32), I can rewrite (15) as follows:

$$\Pi = \beta\delta \left[(1-n) \frac{u'(q_{+1})}{c'\left(\frac{1-n}{n} q_{+1}\right)} + n \right], \tag{35}$$

where $q^s = [(1-n)q]/n$ from (29). I define $\bar{q}^{IF}_{so}$ as the value of DM goods consumption that satisfies (35). I find that $\bar{q}^{IF}_{so}$ is unique because $u'' < 0$, $c'' \geq 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. If the central bank promised that the gross inflation rate from one period ago to the current period is equal to Π, current DM goods consumption is equal to $\bar{q}^{IF}_{so}$.

Because constraint (8) binds, I can rewrite the left-hand side of (15) in the equilibrium as follows:

$$\frac{\phi p q}{\phi_{+1} p_{+1} q_{+1}} \frac{m_{+1}}{m} = \frac{c'\left(\frac{1-n}{n} q\right) q}{c'\left(\frac{1-n}{n} q_{+1}\right) q_{+1}} \frac{M_{+1}}{M}. \tag{36}$$

Substituting this into (15) and calculating it, I obtain the equilibrium path of the DM goods as follows:

$$c'\left(\frac{1-n}{n} q\right) q = c'\left(\frac{1-n}{n} q_{+1}\right) \frac{q_{+1}\beta\delta}{\gamma} \left[(1-n) \frac{u'(q_{+1})}{c'\left(\frac{1-n}{n} q_{+1}\right)} + n \right], \tag{37}$$

where $\gamma \equiv M_{+1}/M$. Since (35) is satisfied, for all periods, $q = \bar{q}^{IF}_{so}$. Therefore, I obtain

$$\gamma = \Pi. \tag{38}$$

From the above discussion, I obtain the following proposition.

Proposition 1. *If the central bank adopts inflation targeting, the equilibrium is characterized by the following values:*

1. *DM goods consumption is given by* $q^b = \bar{q}^{IF}_{so}$ *and DM goods supply is given by* $q^s = \frac{1-n}{n}\bar{q}^{IF}_{so}$,
2. *CM goods consumption is given by* $x = x^*$,

3. The labor supplies of the seller and buyer in the CM are given by

$$h^s = x^* - c'\left(\frac{1-n}{n}\bar{q}_{so}^{IF}\right)\frac{1-n}{n}\bar{q}_{so}^{IF}, \tag{39}$$

$$h^b = x^* + c'\left(\frac{1-n}{n}\bar{q}_{so}^{IF}\right)\bar{q}_{so}^{IF}, \tag{40}$$

4. The gross growth rate of the money supply is given by $\gamma = \Pi$,

5. The gross growth rate of the nominal price of the DM goods, p, is given by $p_{+1}/p = \Pi$.

Proof. I only show points 3 and 5 because the other values are discussed before this proposition.

First, I show that point 3 is correct. Substituting (6), $q^b = \bar{q}_{so}^{IF}$, $q^s = \frac{1-n}{n}\bar{q}_{so}^{IF}$, $x = x^*$, (31), and $M_{+1} = \gamma M$ into (4) and (9), I obtain (39) and (40).

Next, I show point 5 is correct. From (6), I obtain $p = c'(q^s)/\phi$. Since $q^s = \frac{1-n}{n}\bar{q}_{so}^{IF}$ for all t (38) is satisfied, I obtain the gross growth rate of p:

$$\frac{p_{+1}}{p} = \frac{c'\left(\frac{1-n}{n}\bar{q}_{so}^{IF}\right)}{c'\left(\frac{1-n}{n}\bar{q}_{so}^{IF}\right)}\frac{\phi}{\phi_{+1}} = \Pi. \tag{41}$$

□

Next, I obtain the equilibria when the monetary policy rule is nominal growth rate targeting. Substituting (6) and $q^s = \frac{1-n}{n}q$ into (34) and solving the equation for ϕ/ϕ_{+1}, I obtain the following equation:

$$\frac{\phi}{\phi_{+1}} = g\frac{c'\left(\frac{1-n}{n}q\right)q + x^*}{c'\left(\frac{1-n}{n}q_{+1}\right)q_{+1} + x^*}. \tag{42}$$

Substituting (42) into (15), I obtain

$$c'\left(\frac{1-n}{n}q\right)q = \frac{1}{g}\left(c'\left(\frac{1-n}{n}q_{+1}\right)q_{+1} + x^*\right)\beta\delta\left[(1-n)\frac{u'(q_{+1})}{c'\left(\frac{1-n}{n}q_{+1}\right)} + n\right] - x^*. \tag{43}$$

From this equation, I obtain the following proposition.

Proposition 2. *If the central bank adopts nominal growth rate targeting, the positive DM goods consumption in the steady state, $\bar{q}_{so}^{GDP}$, is unique.*

Proof. If the economy is in a steady state and DM goods consumption is positive, I can rewrite (43) as follows:

$$\frac{u'(\bar{q}_{so}^{GDP})}{c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)} = \frac{g/(\beta\delta) - n}{1-n}, \tag{44}$$

where $\bar{q}_{so}^{GDP}$ is DM goods consumption. $\bar{q}_{so}^{GDP}$ is unique because $u'' < 0$, $c'' \geq 0$, $u(0) = \infty$, and $u'(\infty) = 0$. □

I also obtain the following proposition about the stability of the steady state.

Proposition 3. *The steady state is unstable when DM goods consumption in the steady state is equal to $\bar{q}_{so}^{GDP}$.*

Proof. Totally differentiating (43), I obtain

$$\begin{aligned}\frac{dq_{+1}}{dq} &= \frac{g}{\beta\delta}\left[c''\left(\frac{1-n}{n}q\right)\frac{1-n}{n}q + c'\left(\frac{1-n}{n}q\right)\right] \\ &\times \Bigg[\left\{c''\left(\frac{1-n}{n}q_{+1}\right)\frac{1-n}{n}q_{+1} + c'\left(\frac{1-n}{n}q_{+1}\right)\right\}\left\{(1-n)\frac{u'(q_{+1})}{c'\left(\frac{1-n}{n}q_{+1}\right)} + n\right\} \\ &\quad + \left\{c'\left(\frac{1-n}{n}\right)q_{+1} + x^*\right\}(1-n)\frac{u''(q_{+1})c'\left(\frac{1-n}{n}q_{+1}\right) - u'(q_{+1})c''\left(\frac{1-n}{n}q_{+1}\right)}{\left\{c'\left(\frac{1-n}{n}q_{+1}\right)\right\}^2}\Bigg]^{-1}.\end{aligned} \tag{45}$$

From (43), one of the steady states is $q = q_{+1} = 0$. When the above equation is evaluated near this steady state, I obtain

$$\frac{dq_{+1}}{dq}|_{q=q_{+1}=0} = 0 \tag{46}$$

because $u'(0) = \infty$. In the other steady state, $\phi/\phi_{+1} = g$ from (42). Therefore, from (15), I obtain

$$g = \frac{\phi}{\phi_{+1}} = \beta\delta \left[(1-n) \frac{u'(\bar{q}_{so}^{GDP})}{c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)} + n \right]. \tag{47}$$

Using this relationship, (45) when $q = q_{+1} = \bar{q}_{so}^{GDP}$ can be rewritten by

$$\frac{dq_{+1}}{dq}|_{q=q_{+1}=\bar{q}_{so}^{GDP}}$$
$$= \left[1 + \left\{ c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)\bar{q}_{so}^{GDP} + x^* \right\} (1-n) \frac{u''(\bar{q}_{so}^{GDP})c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right) - u'(\bar{q}_{so}^{GDP})c''\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)}{c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)\left[c''\left(\frac{1-n}{n}\bar{q}\right)^2 \frac{1-n}{n}\bar{q}_{so}^{GDP} + c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)\right]} \right]^{-1}. \tag{48}$$

Because $u' > 0$, $c' > 0$, $u'' < 0$ and $c'' \geq 0$, $u''(\bar{q}_{so}^{GDP})c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right) - u'(\bar{q}_{so}^{GDP})c''\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right) < 0$. Because the steady state is unique from Proposition 8, I can draw Figure 1. Therefore,

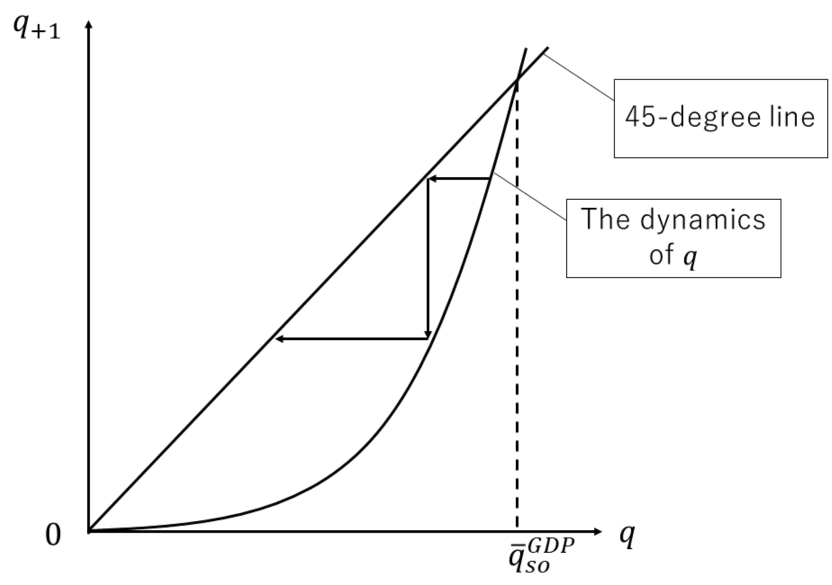

Figure 1: Phase diagram of q

the steady state is unstable when $q = q_{+1} = \bar{q}_{so}^{GDP}$. □

This proposition shows that there are two equilibria at which the economy converges to $q = 0$

and initially jumps to the steady state at which q is positive. Hereafter, I assume that the latter equilibrium is selected and obtain the following proposition.

Proposition 4. *If the central bank adopts nominal growth rate targeting, the equilibrium is characterized by the following values:*

1. *DM goods consumption is given by* $q^b = \bar{q}_{so}^{GDP}$ *and DM goods supply is given by* $q^s = \frac{1-n}{n}\bar{q}_{so}^{GDP}$*,*

2. *CM goods consumption is given by* $x = x^*$*,*

3. *The labor supplies of the seller and buyer in the CM are given by*

$$h^s = x^* - c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)\frac{1-n}{n}\bar{q}_{so}^{GDP}, \tag{49}$$

$$h^b = x^* + c'\left(\frac{1-n}{n}\bar{q}_{so}^{GDP}\right)\bar{q}_{so}^{GDP}, \tag{50}$$

4. *The gross inflation rate is given by* $\phi/\phi_{+1} = g$*,*

5. *The gross growth rate of the nominal price of the DM goods,* p*, is given by* $p_{+1}/p = g$*,*

6. *The gross growth rate of money is given by* $\gamma = g$*.*

Proof. I only show point 6 because the other values are discussed before this proposition or found by using the same method as in Proposition 1.

Since I assume that $pq = M$ for all t, $(p_{+1}q_{+1})/(pq) = M_{+1}/M = \gamma$. Therefore, the following equation is satisfied in the steady state:

$$\gamma = \frac{p_{+1}\bar{q}_{so}^{GDP}}{p\bar{q}_{so}^{GDP}} = \frac{p_{+1}}{p} = g. \tag{51}$$

□

From Propositions 1 and 4, I find that the central bank achieves the target inflation rate or target nominal growth rate, and monetary policy is time-consistent although the agents have a time-inconsistent preference. This is because the sophisticated agents correctly predict their behavior. Then, the agents do not change their behavior from the pre-planned behavior, and the central bank does not need to change its monetary policy plan.

4.4 The equilibrium path in the naïve agent economy

From Assumption 1, all the agents behave the same. Therefore, the expected market-clearing conditions are denoted by

$$(1-n)q_{na}^b = nq_{na}^s, \tag{52}$$

$$x^* = (1-n)h_{na}^b + nh_{na}^s, \tag{53}$$

$$m_{na} = M_{na}. \tag{54}$$

Since the above conditions and $q_{na}^b = m_{na}/p_{na}$ are satisfied, I can define the equilibria as follows.

Definition 3. *The equilibria in the naïve agent economy consist of the quantities* $\{q, x, q_{na}, x_{na}, m_{+1}, m_{na,+1}, h, h_{na}\}$ *and prices* $\{\phi, \phi_{na}, p, p_{na}\}$*, which satisfy the following conditions:*

1. *The optimal conditions of the agent's behavior are given by* (6)*,* (7)*, and* (28)*.*

2. *The optimal conditions of the expected agent's behavior are given by* (19)*,* (20)*, and* (27)*.*

3. *The market-clearing conditions are given by* (29)*–*(31)*.*

4. *The expected market-clearing conditions are given by* (52) *and* (53)*.*

5. *Constraint* (8) *binds.*

First, I discuss the case of inflation targeting. If the central bank adopts inflation targeting, the following equation is satisfied from (27):

$$\Pi = \delta \left[(1-n) \frac{u'(\bar{q}^{IF}_{na,long})}{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,long}\right)} + n \right], \tag{55}$$

where $\bar{q}^{IF}_{na,long}$ is the "long-run" expected DM goods consumption which satisfies (55)[3]. Because (27) is satisfied after two periods from the current period, I call $\bar{q}^{IF}_{na,long}$ the long-run expected DM goods consumption. Following this definition, we can rewrite (27) in the case of inflation targeting as follows:

$$c'\left(\frac{1-n}{n} q^{IF}_{na,long}\right) q^{IF}_{na,long} = c'\left(\frac{1-n}{n} q^{IF}_{na,long}\right) \frac{q^{IF}_{na,long}\delta}{\gamma^{IF}_{na,long}} \left[(1-n) \frac{u'(q^{IF}_{na,long})}{c'\left(\frac{1-n}{n} q^{IF}_{na,long}\right)} + n \right], \tag{56}$$

where $\gamma^{IF}_{na,long}$ is the long-run expected gross growth rate of money. Substituting (55) into this equation and solving it for $\gamma^{IF}_{na,long}$, I obtain

$$\gamma^{IF}_{na,long} = \Pi \tag{57}$$

Next, I discuss the "short-run" expected DM goods consumption, $\bar{q}^{IF}_{na,short}$. Because $\phi/\phi_{na,+1} = \Pi$, I obtain $\bar{q}^{IF}_{na,short}$ from (28), which satisfies the following equation:

$$\Pi = \beta\delta \left[(1-n) \frac{u'(\bar{q}^{IF}_{na,short})}{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,short}\right)} + n \right]. \tag{58}$$

From (27), the following relationship must be satisfied:

$$c'\left(\frac{1-n}{n} q^{IF}_{na,short}\right) q^{IF}_{na,short} = c'\left(\frac{1-n}{n} q^{IF}_{na,long}\right) \frac{q^{IF}_{na,long}\delta}{\gamma^{IF}_{na,short}} \left[(1-n) \frac{u'(q^{IF}_{na,long})}{c'\left(\frac{1-n}{n} q^{IF}_{na,long}\right)} + n \right], \tag{59}$$

[3] $\bar{q}^{IF}_{na,long}$ is unique because $u'' < 0$, $c'' \geq 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. This uniqueness means that $\bar{q}^{IF}_{na,long}$ is constant because Π does not change over time.

where $\gamma_{na,short}$ is the short-run expected gross growth rate of money. Substituting (55) into this equation and solving it for $\gamma_{na,short}$, I obtain

$$\gamma_{na,short}^{IF} = \Pi \frac{c'\left(\frac{1-n}{n} q_{na,long}^{IF}\right) q_{na,long}^{IF}}{c'\left(\frac{1-n}{n} q_{na,short}^{IF}\right) q_{na,short}^{IF}}. \tag{60}$$

Next, I discuss current DM goods consumption, $\bar{q}_{na}^{IF}$. I can rewrite (28) as (56):

$$c'\left(\frac{1-n}{n}\bar{q}_{na}^{IF}\right)\bar{q}_{na}^{IF} = c'\left(\frac{1-n}{n} q_{na,short}^{IF}\right)\frac{q_{na,short}^{IF}\beta\delta}{\gamma}\left[(1-n)\frac{u'(q_{na,short}^{IF})}{c'\left(\frac{1-n}{n} q_{na,short}^{IF}\right)} + n\right]. \tag{61}$$

$\bar{q}_{na}^{IF}$ is determined to satisfy (61). Then, I obtain the following proposition.

Proposition 5. *If the central bank adopts inflation targeting and determines γ, the equilibrium is characterized by the following values:*

1. *Realized, short-run expected, and long-run expected DM goods consumption are given by $\bar{q}_{na}^{IF}$, $\bar{q}_{na,short}^{IF}$ and $\bar{q}_{na,long}^{IF}$, while the realized, short-run expected, and long-run expected DM goods supply are given by $\frac{1-n}{n}\bar{q}_{na}^{IF}$, $\frac{1-n}{n}\bar{q}_{na_short}^{IF}$, and $\frac{1-n}{n}\bar{q}_{na,long}^{IF}$.*

2. *Realized and expected CM goods consumption is given by $x = x_{na} = x^*$,*

3. *The realized, short-run expected, and long-run expected labor supply of the seller and buyer in the CM are given by substituting $\bar{q}_{na}^{IF}$, $\bar{q}_{na,short}^{IF}$ and $\bar{q}_{na,long}^{IF}$ into the following equations:*

$$h^s = x^* - c'\left(\frac{1-n}{n}q\right)\frac{1-n}{n}q,$$
$$h^b = x^* + c'\left(\frac{1-n}{n}q\right)q,$$

4. *The long-run expected gross growth rate of the money supply is given by $\gamma_{na,long} = \Pi$ and the short-run expected gross growth rate of the money supply is given by $\gamma_{na,short} = \Pi \frac{c'\left(\frac{1-n}{n} q_{na,long}^{IF}\right) q_{na,long}^{IF}}{c'\left(\frac{1-n}{n} q_{na,short}^{IF}\right) q_{na,short}^{IF}}$,*

5. *The long-run expected gross growth rate of the nominal price of the DM goods is given by* $p^{IF}_{long,+1}/p^{IF}_{long} = \Pi$ *and the short-run expected gross growth rate of the nominal price of the DM goods is given by* $p^{IF}_{long,+1}/p^{IF}_{short} = \Pi\frac{c'\left(\frac{1-n}{n}q^{IF}_{na,long}\right)}{c'\left(\frac{1-n}{n}q^{IF}_{na,short}\right)}$.

Proof. I only show point 5 because the other values are discussed before this proposition or found by using the same method as in Proposition 1.

From (6), I obtain $p = c'(q^s)/\phi$. Since the long-run expected DM goods consumption is $q^s_{nalong} = \frac{1-n}{n}\bar{q}^{IF}_{na,long}$ for all t , I obtain the gross growth rate of p:

$$\frac{p_{+1,long}}{p_{long}} = \frac{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,long}\right)}{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,long}\right)}\frac{\phi_{na}}{\phi_{na,+1}} = \Pi. \tag{62}$$

Because $p = c'(q^s)/\phi$, the short-run expected, long-run expected DM goods consumption are given by $\bar{q}^{IF}_{na,short}$ and $\bar{q}^{IF}_{na,long}$, and $\phi/\phi_{na,+1} = \Pi$, I obtain the following equation:

$$\frac{p_{+1,long}}{p_{short}} = \frac{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,long}\right)}{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,short}\right)}\Pi. \tag{63}$$

□

From this proposition, I do not obtain the current growth rate of money γ. Therefore, I have to determine the policy rule to determine γ. I consider two cases. The first case is the central bank knows that $q^b = \bar{q}^{IF}_{na}$ for all t and uses γ to induce the realized gross inflation rate to Π. From constraint (8), I obtain $p_{+1}/p = \gamma$. Combining this equation and the seller's first-order condition (6), I also obtain $\phi/\phi_{+1} = \gamma$. Therefore, I obtain the following proposition.

Proposition 6. *If the central bank knows that* $q^b = \bar{q}^{IF}_{na}$ *for all* t *and uses* γ *to induce the realized gross inflation rate to* Π, $\gamma = \Pi$.

Proof. Because the gross realized inflation rate is ϕ/ϕ_{+1} is equal to Π and γ, $\Pi = \gamma$. □

However, the central bank may not know that $q^b = \bar{q}^{IF}_{na}$ for all t. In other words, it predicts that the agent's behavior is given by Proposition 5. Therefore, I consider the second case in which the central bank does not know that $q^b = \bar{q}_{na}^{IF}$ for all t. It adjusts the money supply to maximize the agents' welfare. One of the most likely welfare functions is the sum of the lifetime utility of the sellers and buyers. However, I do not have to consider the future agents' behavior because this is given by Proposition 5. Hence, the welfare function considered by the central bank is the sum of the current instant utility of the sellers and buyers as follows:

$$W \equiv (1-n)u(q) - nc\left(\frac{1-n}{n}q\right), \tag{64}$$

where I omit utility from the CM because goods consumption and the labor supply in the CM do not depend on monetary policy. Then, I obtain the following lemma.

Lemma 1. *If $q = q^*$, (64) is maximized.*

Proof. The first-order condition of (64) is given by

$$u'(q) = c'\left(\frac{1-n}{n}q\right). \tag{65}$$

Since $\phi p = c'(q^s)$ from (6), (65) means that $q = q^*$. Therefore, I obtain this proposition.

□

If the purpose of the central bank is to induce q to q^*, I obtain the following proposition.

Proposition 7. *1. To induce q to q^*, the central bank determines the gross growth rate as follows:*

$$\gamma_{na}^{IF} = \frac{c'\left(\frac{1-n}{n}\bar{q}_{na,short}^{IF}\right)\bar{q}_{na,short}^{IF}}{c'\left(\frac{1-n}{n}q^*\right)q^*}\Pi. \tag{66}$$

2. $\gamma_{na}^{IF} < \Pi$.

Proof. I show that point 1 is correct. Substituting $q_{na}^{IF} = q^*$ and (58) into (61) and solving for γ, I obtain (66).

Next, I show point 2 is correct. From (55) and (58), $\bar{q}^{IF}_{na,short} < \bar{q}^{IF}_{na,long}$ because $\beta < 1$. The maximized q is q^*. Therefore, $\bar{q}^{IF}_{na,short} < \bar{q}^{IF}_{na,long} \leq q^*$. Because this inequality means that $\frac{c'\left(\frac{1-n}{n}\bar{q}^{IF}_{na,short}\right)\bar{q}^{IF}_{na,short}}{c'\left(\frac{1-n}{n}q^*\right)q^*}$ is smaller than 1, $\gamma_{op} < \Pi$. □

This proposition means that the target of the inflation rate is not achieved because γ^{IF}_{na} deviates from the condition of Proposition 6. It is easier to obtain information on the history of the inflation rate than to obtain information on consumption. Therefore, the agents may not believe the central bank's announcement, and the central bank cannot adopt the monetary policy which maximizes (64). If the naïve agents do not doubt the central bank's announcement, the central bank can adopt the monetary policy. This discussion means whether the optimal policy can be realized depends on how people form their expectations. Therefore, inflation targeting is an unstable monetary policy rule in the naïve agent economy.

Second, I discuss nominal growth rate targeting. As in (42), the expected inflation rate is given by

$$\frac{\phi_{na}}{\phi_{na,+1}} = g\frac{c'\left(\frac{1-n}{n}q_{na}\right)q_{na} + x^*}{c'\left(\frac{1-n}{n}q_{na,+1}\right)q_{na,+1} + x^*}. \tag{67}$$

Substituting this equation into (27), I obtain

$$c'\left(\frac{1-n}{n}q_{na}\right)q_{na} = \frac{1}{g}\left(c'\left(\frac{1-n}{n}q_{na,+1}\right)q_{na,+1} + x^*\right)\delta\left[(1-n)\frac{u'(q_{na,+1})}{c'\left(\frac{1-n}{n}q_{na,+1}\right)} + n\right] - x^*. \tag{68}$$

From this equation, I obtain the following proposition.

Proposition 8. *If the central bank adopts nominal growth rate targeting, the positive DM good consumption in the steady state, $\bar{q}^{GDP}_{na,long}$, is unique,*

Proof. If the economy is in a steady state and DM goods consumption is positive, I can rewrite (68) as follows:

$$\frac{u'(\bar{q}^{GDP}_{na,long})}{c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na,long}\right)} = \frac{g/(\delta)-n}{1-n}, \tag{69}$$

where $\bar{q}^{GDP}_{na,long}$ is expected DM goods consumption under nominal growth rate targeting. $\bar{q}^{GDP}_{na,long}$ is unique because $u'' < 0$, $c'' \geq 0$, $u(0) = \infty$, and $u'(\infty) = 0$. □

From this proposition and (27), I obtain

$$\frac{\phi_{na}}{\phi_{na,+1}} = \delta\left[(1-n)\frac{u'(\bar{q}^{GDP}_{na,long})}{c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na,long}\right)} + n\right] = g. \tag{70}$$

Substituting this equation into (28), I obtain

$$\frac{\phi}{\phi_{na,+1}} = \beta g. \tag{71}$$

From this equation, and (42), I obtain

$$c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na}\right)\bar{q}^{GDP}_{na} = \beta\left(c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na,long}\right)\bar{q}^{GDP}_{na,long} + x^*\right) - x^*. \tag{72}$$

From this equation, I find that $\bar{q}^{GDP}_{na}$ is also constant in the steady state because $\bar{q}^{GDP}_{na,long}$ is constant. The other variables are obtained as in the following proposition.

Proposition 9. *If the central bank adopts nominal growth targeting and determines g, the equilibrium is characterized by the following values:*

1. *Realized and expected DM goods consumption are given by $\bar{q}^{GDP}_{na}$ and $\bar{q}^{GDP}_{na,long}$ and the current, short-run expected, and long-run expected DM goods supply are given by $\frac{1-n}{n}\bar{q}^{GDP}_{na}$ and $\frac{1-n}{n}\bar{q}^{GDP}_{na,long}$,*

2. *Realized and expected CM goods consumption is given by $x = x_{na} = x^*$,*

3. *The realized and expected labor supply of the seller and buyer in the CM are given by substituting* $\frac{1-n}{n}\bar{q}^{GDP}_{na}$ *and* $\bar{q}^{GDP}_{na,long}$ *into the following equations:*

$$h^s = x^* - c'\left(\frac{1-n}{n}q\right)\frac{1-n}{n}q,$$
$$h^b = x^* + c'\left(\frac{1-n}{n}q\right)q,$$

4. *The realized and long-run expected gross inflation rates are given by* $\phi/\phi_{+1} = \phi_{na}/\phi_{na,+1} = g$ *and the short-run expected gross inflation rate is given by* $\phi/\phi_{na,+1} = \beta g$

5. *The realized and long-run expected gross growth rates of the nominal price of the DM goods are given by* $p_{+1}/p = p_{na,+1}/p_{na} = g$ *and the short-run expected gross growth rate of the nominal price of the DM goods is given by* $p_{na,+1}/p = \frac{c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na,long}\right)}{c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na}\right)}\beta g$*,*

6. *The realized and long-run expected gross growth rates of money are given by* $\gamma = \gamma_{na,long} = g$ *and the short-run expected gross growth rate of money is given by* $\gamma_{na,short} = \frac{c'\left(\frac{1-n}{n}\bar{q}^{GDP}_{na,long}\right)\bar{q}^{GDP}_{na,long}}{c'\left(\frac{1-n}{n}q^{GDP}_{na}\right)q^{GDP}_{na}}\beta g$*.*

Proof. This proposition is shown by the discussion before this proposition or found by using the same method as in Proposition 1, Proposition 4, or Proposition 5. Therefore, I do not show that this proposition is correct. □

This proposition shows that the realized nominal growth rate is consistent with that expected under nominal growth rate targeting. Therefore, nominal growth rate targeting is a trusted policy rule by the naïve agents because it has no time inconsistency. This point differs from the case of inflation targeting, which can only bind the expected variables (i.e., consumption and the growth rate of money), even though nominal growth rate targeting can bind both the expected and the current variables. Therefore, the central bank can determine and change the current money supply from the past plan to adjust current consumption to the optimal level under inflation targeting. This difference does not happen in the sophisticated agent economy because the agent's behavior is time-consistent even though the agent has a time-inconsistent preference. Therefore, the central bank in the sophisticated agent economy does not need to change its pre-determined policy to adjust the agent's time inconsistency.

5 Conclusion

I have extended the monetary search model in which agents have present bias given by hyperbolic discounting. Through this model, in the naïve agent economy, I have shown that inflation targeting may be a time-inconsistent monetary policy rule because the target of the inflation rate is not achieved by the central bank which is a welfare maximizer, even though nominal growth rate targeting is a time-consistent rule. This result implies that inflation targeting cannot play a role in stabilizing the agent's expectation. Therefore, nominal growth rate targeting is a better rule than inflation targeting if we emphasize this role in the naïve agent economy. I have also shown that this difference is not found in the sophisticated agent economy. This implies that inflation targeting can be adopted if the sophisticated agent economy can be distinguished from the naïve agent economy. However, as both types of agents exist in the real-world economy, this is impossible. Hence, there is no positive reason why the central bank adopts inflation targeting under agents' time inconsistency.

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