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This paper examines the policy effects of expanding formal child-care coverage on family fertility and education decisions in an overlapping generations model with Nash bargaining between sexes. The effects depend on a mother's education level: If it is sufficiently high (low), then the policy lowers (raises) fertility and raises (lowers) educational investment in children in the long term. There is an intermediate case in which the policy raises both the fertility rate and educational investment in daughters but lowers investment in sons. The policy also raises the probability of marriage and child bearing by lowering the cost of having children.

Keywords: Nash bargaining; fertility; educational investment; child-care policy
JEL Classifications: D91; H53; J13; J16

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1. Introduction

Recently, it has been demonstrated that the persistent fertility decline in some high-income developed countries have turned up at higher income levels around 2000, producing higher fertility (e.g., Apps and Rees, 2004; Feyrer et al., 2008; Myrskylä et al., 2009; Day, 2012, 2016; Luci-Greulich and Thévenon, 2013, 2014; Hazan and Zoabi, 2015). One reason for such a fertility rebound is regarded as family policies undertaken in these countries. Janta (2014), among others, reported that family policies related to child care vary among the EU countries. Luci-Greulich and Thévenon (2013) used macro panel data from 18 OECD countries obtained during 1982–2007 to show that provision of child-care services to families with children under age three and in-cash benefits covering childhood are more effective as child-care policies than leave entitlements, benefits granted at childbirth, and cash benefits provided to children under age twenty.

On the hand, The OECD (2012) reported that the likelihood that young people will enter a university program increased dramatically from 47% to 59% in OECD countries during 2000–2009 and that women are driving these increases, i.e., the proportion of women expected to enter a university rose from 51% in 2000 to 66% in 2009. Currie and Moretti (2003) showed that, in the US, female average educational attainment and its returns, i.e., female wages, have increased tremendously over the past decades, and Goldin et al. (2006) reported that the proportion of women who graduated from a four-year college have exceeded those for men in the US in 2003. The purpose of the present study is to provide a theoretical basis explaining these facts consistently and then to examine the effects of a child-care policy. This study uses an overlapping generations model in which family decisions on fertility and educational investment in children are made through Nash bargaining between women and men, including whether to marry or not.

This study focuses on formal child care and analyzes the effects of an expansion of its coverage for children as a child-care policy. Although couples, especially mothers, must care for their children for a certain period at home, i.e., at least, during pregnant, delivery, and lactation periods, children who are older than a certain age are expected to have formal child care outside the home.[1] However, as Janta (2014) reported, the EU's Barcelona target of the provision of formal child care for 33% of children under age three had been achieved in only 10 out of 28 EU countries. The target of 90% coverage of children from age three to minimum compulsory school age had been achieved in only

[1] Iyigun and Walsh (2007a), among others, took it as granted that the inherent biological differences between sexes in the requirements of parental time investment.

11 out of 28 EU countries in 2010. The coverage of formal child care is still rather low in these EU countries, but the ratio of expenditure on early child education and care to GDP has increased in countries such as France and the UK since about 2000.[2]

Improved accessibility and coverage of formal child care lower the cost of mothers' child bearing by shortening the child-bearing time at home, but they increase the family wage income by enabling mothers to work longer in the labor market. Although the former effect tends to increase the number of children, the latter might increase educational expenditure on their children, thereby reducing the number of children via the quantity-quality tradeoff of children. Therefore, expansions of formal child-care coverage might not necessarily raise the fertility rate. The latter effect renders the mothers' contribution to family income greater. Consequently, parents might raise educational investment in girls at the expense of that for boys.

Family decision-making related to fertility and children's education will be undertaken by family bargaining between women and men.[3] Such bargaining in families has been analyzed by, for example, Manser and Brown (1980), McElroy and Horney (1981), Chiappori (1988, 1992) and Apps and Rees (1997), but their analyses are mostly static.[4] Recently, Echevarria and Merlo (1999) developed the work of Manser and Brown (1980) and McElroy and Horney (1981) to a dynamic environment and

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[2] See, for example, OECD Statistics (Social Expenditure, Public expenditure on family by type of expenditure, in % GDP, http://stats.oecd.org/, 10 June 2015). Another example is the limited accessibility to formal child care. For example, in Japan, the ratio of the number of children on a waiting list to use day nurseries ("Taiki-Jido" in Japanese) to the total number of children aged 0 to 4 was high, i.e., about 0.81%, in 2014 (Source: Ministry of Health, Labour and Welfare: http://www.mhlw.go.jp/stf/houdou/0000078441.html and National institute of Population and Social Security Research: http://www.ipss.go.jp/shoshika/tokei/Populr/P_Detail2016.asp?fname=T02-02.htm, both cited on 15 June 2016).

[3] Although non-cooperative factors in family bargaining have also been suggested in the literature (e.g., Lundberg and Pollak, 1993; Konrad and Lommerud, 2000; Basu, 2006; Iyigun and Walsh, 2007a), we assume away such behaviors in this paper. The threat point in the present study is 'being unmarried' in a cooperative game as in Echevarria and Merlo (1999), whereas Konrad and Lommerud (2000) and Basu (2006) examined household's decisions in two-stage games by incorporating non-cooperative choices of human capital investment into a collective decisions of family public goods (e.g., the number of children). For the difference between a collective approach and Nash bargaining, see Basu (2006). Bergstrom (1996) provides a survey on the development of the economics of the family.

[4] It is widely perceived that unitary models are not supported empirically. See, for example, Lundberg et al. (1996) and Blundell et al. (2007). There are two strands in the literature: Manser and Brown (1980) and McElroy and Horney (1981) are Nash bargaining models, whereas Chiappori (1988, 1992) and Apps and Rees (1997) take the collective approaches.

analyzed the process of intra-household decision-making in an intergenerational dynamic household Nash-bargaining model, although they did not examine the long-term time path of family decisions. Basu (2006) built a dynamic model based on a collective approach as an apt description of a single period to examine a stationary point of this process as the equilibrium. Iyigun and Walsh (2007a) used an overlapping generation setting to examine the results of two-stage bargaining on fertility and education. In Iyigun and Walsh's collective model, the bargaining powers of women and men depend on their respective wage earnings, which are in turn determined by their own educational choices.[5] More recently, Doepke and Tertilt (2009) examined the dynamic effects of female empowerment. However, their focus is a regime transition from a patriarchy regime, under which men make all family decisions, to the empowerment regime, under which both spouses have equal bargaining power related to household decisions.[6] No studies cited above examined policy effects on the dynamics of family bargaining.[7]

The present study specifically examines the policy effects on dynamics of bargained decisions of marriage, fertility and offspring's education between women and men, by introducing the policy of formal child-care services provision into the over-lapping-generations Nash-bargaining model of Echevarria and Merlo (1999).[8] The main results are the following. The policy effect depends on the mother's education level at the time of policy change. If a mother's education level is sufficiently low, then the expansion policy of the coverage of public child care will raise the fertility rate and lower educational investment in children. In contrast, if a mother's education level is sufficiently high, then the expansion of child-care coverage can lower the fertility rate and raise educational investment in both daughters and sons. In this case, because mothers can earn high wage income using child-care services, they are expected to reduce the number of children and invest more in each child's human capital, as

[5] Using a two-period model, Kemnitz and Thum (2015) also demonstrated that endogenous bargaining power leads to a systematic downward bias in fertility under efficient bargaining.

[6] De la Croix and Donckt (2010) also examined the effect of empowering women on fertility in an overlapping generations model with nonunitary households. They described that, in order to ease economies out of the corner regime with high fertility and no female market labor, it is necessary to promote female survival probability and infant survival rates.

[7] Doepke and Tertilt (2009: p. 1554) implicitly assumed that the government can set up bargaining power of spouses through political regimes. In this sense, they considered policy.

[8] Komura (2013), assuming social norms related to the number of children, examined the relation between the relative bargaining power of women and fertility.

asserted in arguments related to the quantity-quality tradeoff of children. These effects are typically expected to occur from such a child-care policy change.

Most notably, an intermediate case exists: When a mother's education level is not too high and not too low, the expanded child-care provision raises both the fertility rate and educational investment in daughters. The tradeoff between the quantity and quality of children can be mitigated by the policy. The policy lowers educational investment in sons although the education level of boys is still higher than that of daughters. In this intermediate case, the policy change can simultaneously enable parents to have more children. Moreover, it induces mothers to work longer in the market, earnings a higher wage income. Although having more children tends to shorten the mother's working time slightly, the expanded child-care provision frees mothers from child-rearing at home. It might increase the mother's labor supply, more than offsetting the negative effect. In such a case, greater educational investment in girls will result in higher wage income to their household. Therefore, parents can expect that investing more in daughters and even less in sons might bring greater wage income to the children's families. In other words, with the policy change, the bargaining that occurs within a couple induces them to increase educational investment in daughters at the cost of investment in sons in this case, though increasing the number of children. Not only is the last result not obtained in the so-called unitary model, it is also not common in collective models. Furthermore, results show that a formal child-care expansion policy might induce women and men to get married.

The structure of the paper is the following. The next section introduces the model and examines the long-term equilibrium. Section 3 examines the policy effects analytically and then presents a numerical example. Using parameters commonly found in the literature, we obtain the intermediate case. The last section presents conclusions reached through these analyses.

2. The Model

The model is an overlapping generations model that includes intergenerationally pure-altruistic individuals of two types: women and men. Each individual lives for two periods: childhood and adulthood. Daughters and sons are fed and educated by their parents in the first period: childhood. Men work in the labor market throughout the second period while women allocate their time between working in the labor market and, if married, rearing children at home in the second period: adulthood. Following Echevarria and Merlo (1999), we assume that women and men have the same preferences over their own consumption, the number of children, and their children's well-being.[9] Both women and men face the same wage schedule conditional on the education they received. We assume that each has only one chance of getting married and assume that the matching is random. We also assume that the bargaining covers the term of binding prenuptial agreements specifying consumption allocation between themselves during the marriage, the number of children, and educational investments in daughters and sons.[10] The government provides child-care services, financing them through lump-sum taxes on individuals.

2.1. Individuals

Individuals decide whether to get married when they become adults. If they get married, then they determine the number of children and intra-household allocation of their combined income among consumption and children's education through bargaining. We assume that each couple has an equal number of daughters and sons to avoid the issue of matching individuals. If not married, then they consume their wage income after paying taxes. We employ a Nash bargaining solution to the negotiation problem, for which the threat point is given by the utility level that the individual can achieve when not married.

We assume that women must spend a certain period of time to care for each child because of maternal and lactation periods if married. However, we assume away the mother's utility from child rearing at home.[11] If the required total rearing time per child is constant and denoted by $z > 0$, then the rearing time of n_t children for the mother is zn_t. For our analytical purpose, we assume that a fraction $1-\sigma$ of child-rearing burden of mothers is covered by formal child-care services ($0 < \sigma \leq 1$). Therefore, the mother's child-rearing time at home is given as $\sigma z n_t$.

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[9] Although Doepke and Tertilt (2009) asserted that there is a substantial empirical literature supporting that mothers and fathers have different views on the tradeoff (therefore, based on the collective approach), we assume that they have the same view in order to employ the Nash bargaining solution, which requires symmetry for a unique solution to exist.

[10] After surveying the literature related to developing countries, Mason and Taj (1987) concluded that the gender difference in fertility goals tends to become smaller under relatively modern, gender-equal conditions.

[11] This assumption might not hold in reality, although it is usually made in the literature (e.g., Iyigun and Walsh, 2007a). Parents are assumed to obtain utility from having children and from being with them before they leave home.

We assume that there is a continuum of individuals, half of whom are women and the rest of whom are men in the initial period. The von-Neumann-Morgenstern utility of a representative individual of gender i in period t is given as

(1) $$U_{it} = \begin{cases} c_{it} + u(n_t) + \frac{\gamma}{2}(U_{mt+1} + U_{ft+1}) & \text{if married} \\ c_{it} & \text{if not married} \end{cases}$$

where c_{it} denotes the consumption of an adult of gender i ($i = m$ or f, i.e., male or female) and where $u(n_t)$ is the utility from having n_t children, $u'(.) > 0$ and $u''(.) < 0$. $\gamma \in (0,1)$ denotes the degree of altruism.

Normalizing the length of a period to unity and denoting the wage function as $w(e_{it})$, where $w'(.) > 0$ and $w''(.) < 0$ where e_{it} denotes the education level received during childhood, the wage income of men in the adulthood period is given as $w(e_{mt})$. However, the wage income of a woman during the adulthood period is $w(e_{ft})$ if she stays single, whereas it is $(1-\sigma z n_t)w(e_{ft})$ if she gets married (and has n_t children). Letting the price of education be constant at p, then the budget constraint during the adulthood of a couple is given as

(2) $$w(e_{mt}) + (1-\sigma z n_t)w(e_{ft}) - T_t = c_{mt} + c_{ft} + \frac{p(e_{mt+1} + e_{ft+1})n_t}{2},$$

where $T_t/2$ is per capita tax in period t, e_{ft+1} and e_{mt+1} respectively represent the couple's educational investment in daughters and sons.[12]

Each couple solves the following Nash bargaining problem:

(3) $$\underset{\substack{c_{mt}, c_{ft}, n_t, \\ e_{mt+1}, e_{ft+1}}}{Max} \{c_{mt} + u(n_t) + \frac{\gamma}{2}[V^m(e_{mt+1}, \tilde{e}_{ft+1}) + V^f(e_{ft+1}, \tilde{e}_{mt+1})] - [w(e_{mt}) - \frac{T_t}{2}]\}$$

$$\cdot \{c_{ft} + u(n_t) + \frac{\gamma}{2}[V^m(e_{mt+1}, \tilde{e}_{ft+1}) + V^f(e_{ft+1}, \tilde{e}_{mt+1})] - [w(e_{ft}) - \frac{T_t}{2}]\}$$

such that

(4) $$w(e_{mt}) + (1-\sigma z n_t)w(e_{ft}) - T_t = c_{mt} + c_{ft} + \frac{p(e_{mt+1} + e_{ft+1})n_t}{2},$$

[12] The price is taken to be pegged to the goods price, which is assumed to be unity.

where

(5) $$V^i(e_{mt}, e_{ft}) = c_{it} + u(n_t) + \frac{\gamma}{2}[V^m(e_{mt+1}, \tilde{e}_{ft+1}) + V^f(e_{ft+1}, \tilde{e}_{mt+1})]$$

$(i = m, f)$ is the utility an individual of gender i obtains from marriage, given their own and their spouse's levels of education and taxation. A solution to problem (3) is a pair of value functions V^m and V^f, where $V^i(e_{it}, \tilde{e}_{jt})$ signifies that when parents of gender i choose the level of educational investment of their children (e_{it+1}), they take the behavior of the parents of their children's future spouses ($\tilde{e}_{jt+1}$) as given and select the best response to it $(i, j = m, f; i \neq j)$. Although the number of children and the levels of educational investment (n_t, e_{ft}, e_{mt}) are the state variables for a family, these variables are not sufficient to describe the decision problem for a couple. Although parents care about the welfare of their daughters and sons, which in turn depends on the education level of their future spouses, the daughters and sons of a given couple do not marry each other. Parents are assumed to draw spouses of their children at random from other families. Therefore, we also need a state variable that summarizes the couple's expectation regarding the education level of their children's spouses. Because all couples are assumed to start out with the same education levels in the present study, these state variables can be described as the economy-wide averages of female and male education levels $(\tilde{e}_{ft}, \tilde{e}_{mt})$, which are always equal to the education level of each, respectively, i.e., $\tilde{e}_{it} = e_{it}$ $(i = m, f)$.[13]

We assume that we can obtain a unique time path of the state variables (e_{ft}, e_{mt}) for the initial levels (e_{f0}, e_{m0}) given the policy (T_t, σ).[14]

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[13] Although Doepke and Tertilt (2009) assumed human capital stock formation from generation to generation with intergenerationally external effects, we assume away the intergenerational externalities.

[14] Nash (1953) showed that, in order for a unique feasible outcome to be selected, a bargaining solution should have the following four properties: (i) Pareto-optimality, (ii) symmetry, (iii) independence of equivalent utility representation, and (iv) independence of irrelevant alternatives (see Zhang, 2005). In the present setting, although we cannot rule out the possibility of multiple steady states because condition (iii) might not be

Assuming an interior solution, the first-order conditions for problem (2) give the following equations:

$$c_{mt} = w(e_{mt}) - \frac{\sigma z n_t w(e_{ft}) + T_t + \frac{p(e_{mt+1} + e_{ft+1})}{2} n_t}{2}, \tag{6}$$

$$c_{ft} = w(e_{ft}) - \frac{\sigma z n_t w(e_{ft}) + T_t + \frac{p(e_{mt+1} + e_{ft+1})}{2} n_t}{2}, \tag{7}$$

$$w'(e_{mt+1}) = \frac{p n_{t+1}}{2\gamma}, \tag{8}$$

$$w'(e_{ft+1}) = \frac{p n_{t+1}}{2\gamma(1 - \sigma z n_{t+1})}, \tag{9}$$

$$2u'(n_t) = \sigma z w(e_{ft}) + \frac{p(e_{mt+1} + e_{ft+1})}{2}. \tag{10}$$

Equations (6) and (7) show that the spouse with higher education consumes more. They also show that parents equally pay the cost of rearing children, and their educational investment as well as taxation. As reported by Echevarria and Merlo (1999), these are natural results in a Nash bargaining for a couple. Equations (8) and (9) show that, for the expectation of the Nash bargained number of (grand-)children, educational investment in boys is higher than that in girls, i.e., $e_{mt+1} > e_{ft+1}$. Because men work longer, educational investment in boys will raise their household income under the same wage function.

Equation (10) shows that the quantity-quality tradeoff of children, i.e., n_t and (e_{ft+1}, e_{mt+1}), depends on the mother's education attainment but not on the father's education.[15] As empirical evidence, Black et al. (2003) used administrative registers and census data from Statistics Norway during the 1960s and early 1970s to demonstrate significant causal relationship between a mother's education and her son's

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satisfied, steady states can be shown to be locally stable (see Appendix A).

[15] For example, Currie and Moretti (2003) showed that increases in maternal education over the past 30 years have had strong positive effects on birth outcomes such as birth weights, and the benefits were estimated to be a reduction of $5.5 to $6 billion in health, education, and other costs. Sewell and Shah (1968) asserted that when parents have discrepant levels of educational achievement, which parent's education has more effect on educational aspiration and achievement of children depends on the child's gender and intelligence level as well as on each parent's level of educational achievement.

education, but no relationship between the father's education and his children's education. That study also found no evidence of a tradeoff between the quantity and quality of children as mothers acquire more education.

In equation (10), presuming certain levels of educational investment in children (e_{ft+1}, e_{mt+1}), one can show that when the education level of a mother (e_{ft}) is high, the number of children (n_t) satisfying the equation is lower as long as $u''(.) < 0$. When the education level of a mother is high, the time cost of child rearing at home (i.e., the opportunity cost) is high. The couple, *ceteris paribus* reduces the number of children, thereby increasing the mothers' labor supply, and with greater wage income, increases educational investment in their children. This result is apparently consistent with the conventional argument of the quantity-quality tradeoff of children (e.g., Becker and Lewis, 1973; Barro and Becker, 1989; Becker and Barro, 1988).

2.2. Government

The government provides child-care services and finances them by lump-sum taxes on couples in each period.[16] In the present paper, we assume that publicly provided child-care services are inadequate to substitute at-home child care of mothers completely and that the degree of coverage is described as fraction $1-\sigma$. For analytical simplicity, we implicitly assume that the government provides formal child-care services using female workers.[17] The budget constraint of the government is given as

(11) $$(1-\sigma)zn_t w(e_{ft}) = T_t / \delta_t$$

in per-couple terms, where δ_t denotes the population ratio of the number of married

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[16] We can assume a wage tax instead of lump-sum taxes although the wage tax reduces the opportunity cost of child-rearing cost at home, thereby exerting a positive effect on fertility.

[17] This assumption is not necessarily needed. Although a female worker can care for more than one child, the female workers need to be trained specifically so as to care for the children covered by the policy expansion. We assume that such costs will be financed entirely by taxes. In reality, although all the workers in the (public) child-care service sector might not be necessarily female, the ratio of males is fairly low even in developed countries. For example, see the Japan Gender Equality Bureau Cabinet Office (http//www.gender.go.jp/about_danjo/whitepaper/h26/zentai/html/column/clm_04.html: 1 January 2015) and the Japanese Embassy in Norway (http://www.no.emb-japan.go.jp/Japanese/Nikokukan/nikokukan_files/danjyobyoudou.pdf: 1 January 2015).

individuals to the total population in period t, where $0 < \delta_t \leq 1$.

2.3. Dynamic system

Because individuals are assumed to be identical except for gender, we might assume that all individuals get married, i.e., $\delta_t = 1$, at this point in discussion.[18] Inserting T_t from (11) into (6) and (7), we can obtain a dynamic system that is given by the following equations:

$$c_{mt} = w(e_{mt}) - \frac{zn_t w(e_{ft}) + \frac{p(e_{mt+1} + e_{ft+1})}{2} n_t}{2}, \tag{12}$$

$$c_{ft} = w(e_{ft}) - \frac{zn_t w(e_{ft}) + \frac{p(e_{mt+1} + e_{ft+1})}{2} n_t}{2}, \tag{13}$$

and (8), (9) and (10). The system determines $(n_t, c_{mt}, c_{ft}, e_{mt+1}, e_{ft+1})$ for predetermined variables (e_{mt}, e_{ft}) and expectation on n_{t+1}. Assuming that the expectation on fertility is rational, the steady state, if it exists, is definable as (n, c_m, c_f, e_m, e_f) satisfying (1), (8), (9), (12), and (13).

In order to obtain the explicit solution to the dynamics, we specify the wage function as $w(e) = \theta \ln(1+e)$ and the utility from having children as $u(n) = \varepsilon \ln n$ in the following.[19] For analytical simplicity, we assume that $\varepsilon - \theta\gamma < 0$ is satisfied.[20]

Under these assumptions, equations (8)-(10) can be written as

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[18] Otherwise, no individuals get married. For analytical purposes, it is necessary to have at least one married couple.

[19] This wage function means that the wage rate of an individual is zero if his parents did not invest in him. We can instead assume that $w(e) = \theta \ln(a+e)$ where $a > 1$, in which case the wage rate is positive even if his parents did not invest in him.

[20] This assumption implies that an increase in the number of children of a generation will not *ceteris paribus* raise that of the next generation by less than the original increase, i.e., $\partial n_{t+1} / \partial n_t < 1$. It is noteworthy that the violation of the inequality does not *per se* render the results in the present paper invalid. The utility weight on the number of children ε, which can be equal to or greater than the utility weight on the welfare of offspring γ, is commonly used in the literature (e.g., de la Croix and Doepke, 2003).

(14) $$1+e_{mt+1}=\frac{2\gamma\theta}{pn_{t+1}},$$

(15) $$1+e_{ft+1}=\frac{2\gamma\theta}{pn_{t+1}}(1-\sigma z n_{t+1}),$$

(16) $$\frac{2\varepsilon}{n_t}=\sigma z\theta\ln(1+e_{ft})+\frac{p(e_{mt+1}+e_{ft+1})}{2}.$$

Eliminating e_{mt+1} from these three equations, we obtain the simultaneous difference equations of e_{ft+1} and n_t as

(17) $$n_{t+1}=\frac{2}{\frac{1}{\gamma\theta}[\frac{2\varepsilon}{n_t}-\sigma z\theta\ln(1+e_{ft})+p]+\sigma z},$$

(18) $$1+e_{ft+1}=\frac{1}{p}[\frac{2\varepsilon}{n_t}-\sigma z\theta\ln(1+e_{ft})+p]-\frac{\gamma\theta\sigma z}{p}.$$

In the present setting, although both state variables (n_t,e_{ft}) are non-predetermined jump variables in the two-dimensional difference equations, the values of the state variables in each period depend on their values in the previous period, as shown in (17) and (18). In order to examine the dynamics of the system, we draw a phase diagram, which is depicted as Figure 1 (see Appendix A). Assuming the existence of a steady state and examining its stability, one can show that the steady state can be considered to be stable. If the state variables are not those corresponding to a steady state, then the steady state will not be reached in one period and the transition to the steady state will take a number of periods. The stability condition means that

(19) $$p(1+e_f)(\varepsilon-\gamma\theta)<\sigma z\theta^2\gamma<p(1+e_f)(\varepsilon+\gamma\theta).$$

We assume that the stability condition is satisfied in the following.[21]

Up to this point in the discussion, it has been assumed that all individuals get married and have children.[22] The condition guaranteeing marriage is that the benefit of

[21] The steady state is unstable if the second inequality is not satisfied. However, under the assumption of perfect foresight, the system is expected to jump to the steady state if it is unstable. For expositional purposes we assume the inequality in this paper.

[22] The positive number of children derives from the specification of the utility function. Under a general form of utility function, individuals might have no child. See, for example, Hirazawa et al. (2014).

having children is greater than the cost of having children (see Appendix B).

3. Effects of Expanding Formal Child-Care Services

3.1. Long-term effects

Now we examine the effects of an expansion of the coverage of formal child-care services. The policy effect can be described by a smaller fraction of child-rearing time at home (σ). We consider the long-term effect in this sub-section.[23]

Evaluating the variables in the steady state, we obtain the following equation from (17) and (18):

$$(20)\qquad \begin{pmatrix} \dfrac{\varepsilon}{\gamma\theta}-1 & \dfrac{n^2\sigma z}{2\gamma(1+e_f)} \\ -\dfrac{2\varepsilon}{pn^2} & -(1+\dfrac{\sigma z\theta}{p(1+e_f)}) \end{pmatrix} \begin{pmatrix} dn/d\sigma \\ de_f/d\sigma \end{pmatrix} = \begin{pmatrix} \dfrac{n^2 z}{2\gamma}[\gamma-\ln(1+e_f)] \\ \dfrac{\theta z}{p}[\gamma+\ln(1+e_f)] \end{pmatrix}.$$

Letting the determinant of the coefficient matrix on the left-hand side of (20) be

$$(21)\qquad H=\frac{p(1+e_f)(\gamma\theta-\varepsilon)+\sigma z\gamma\theta^2}{\gamma\theta p(1+e_f)},$$

we obtain

$$(22)\qquad \frac{dn}{d\sigma}=H^{-1}\frac{zn^2}{2\gamma}\{[\ln(1+e_f)-\gamma]-\frac{2\sigma z\theta\gamma}{p(1+e_f)}\},$$

$$(23)\qquad \frac{de_f}{d\sigma}=H^{-1}\frac{\theta z}{p}[\frac{2\varepsilon}{\theta}-\gamma-\ln(1+e_f)].$$

Although the sign of H is positive from the stability condition (19), the signs of (22) and (23) cannot be determined *a priori*. Equation (22) implies that

$$(24)\qquad \frac{dn}{d\sigma}\overset{>}{\underset{<}{=}}0 \quad \text{as} \quad \ln(1+e_f)\overset{>}{\underset{<}{=}}\gamma+\frac{2\sigma z\theta\gamma}{p(1+e_f)},$$

that is, if the steady-state education level of women is sufficiently high (low), the expansion of coverage of formal child care will lower (raise) the fertility rate. The effect on the fertility rate is not necessarily positive, depending on the education level of women. This result countervails the expected effect of the policy: When the education level of mothers is sufficiently high, the expanded availability of child care outside the

[23] The transition will be considered in a numerical example in the next sub-section.

home encourages parents to have more children. In other words, it is only at low mother's education levels that the policy raises the fertility rate as expected.

On the other hand, from (23) we obtain that:

(25) $$\frac{de_f}{d\sigma} \overset{>}{\underset{<}{=}} 0 \quad \text{as} \quad \ln(1+e_f) \overset{<}{\underset{>}{=}} \frac{2\varepsilon}{\theta} - \gamma .$$

That is, if the mothers' education level is sufficiently high (low), then the expanded coverage of child care will raise (lower) the steady state education level of daughters and will raise that of mothers as well. When the mothers' education level is low, parents will lower the daughters' education level, although the mothers' working time might lengthen. In contrast, if the mothers' education level is high, then parents will prefer higher female wage rates and raise the daughters' education level.

Using (14), we also obtain

(26) $$\text{sgn}(de_m / d\sigma) = \text{sgn}(-dn / d\sigma) .$$

The change in parents' educational investment in sons has the opposite sign of that in the number of children. The expansion of coverage of formal child care will raise (lower) educational investment in sons if the steady-state education level of mothers is sufficiently high (low).

Paying attention to the difference in the right-hand sides of (24) and (25), we combine the results (24), (25), and (26). From the stability condition, one obtains

(27) $$\left(\frac{2\varepsilon}{\theta} - \gamma\right) - \left(\gamma + \frac{2\sigma z \theta\gamma}{p(1+e_f)}\right) = 2\frac{(\varepsilon - \theta\gamma)p(1+e_f) - \sigma z \theta^2 \gamma}{\theta p(1+e_f)} < 0 ,$$

when $\varepsilon - \theta\gamma < 0$. Therefore, we derive the following result:[24]

Proposition *Assume that a stable steady state of the dynamic system, (17) and (18), exists and that* $\varepsilon - \theta\gamma < 0$ *holds. Then*

(i) $\frac{dn}{d\sigma} < 0$; $\frac{de_f}{d\sigma} \geq 0$ and $\frac{de_m}{d\sigma} > 0$ if $\ln(1+e_f) \leq \frac{2\varepsilon}{\theta} - \gamma$,

(ii) $\frac{dn}{d\sigma} < 0$; $\frac{de_f}{d\sigma} < 0$ and $\frac{de_m}{d\sigma} > 0$ if $\frac{2\varepsilon}{\theta} - \gamma < \ln(1+e_f) < \gamma + \frac{2\sigma z \theta\gamma}{p(1+e_f)}$,

[24] When condition $\varepsilon - \theta\gamma < 0$ is not satisfied, case (ii) of Proposition might disappear. If the absolute value of $\varepsilon - \theta\gamma > 0$ is sufficiently great, then there can be a case (ii') $dn / d\sigma > 0$ and $de_f / d\sigma > 0$ as long as the steady state is stable. Nevertheless, this case seems to be implausible because z is fairly small.

(iii) $\frac{dn}{d\sigma} \geq 0$; $\frac{de_f}{d\sigma} < 0$ and $\frac{de_m}{d\sigma} \leq 0$ if $\gamma + \frac{2\sigma z\theta\gamma}{p(1+e_f)} \leq \ln(1+e_f)$.

It is noteworthy that there is a case in which education levels of daughters and sons can be affected by the policy in mutually opposite directions, as illustrated in case (ii). In this case, parents increase both the number of children and the education level of daughters, but they reduce the education level of boys. At the education level of mothers satisfying $\frac{2\varepsilon}{\theta} - \gamma < \ln(1+e_f) < \gamma + \frac{2\sigma z\theta\gamma}{p(1+e_f)}$, the expansion of public child-care coverage not only shortens mothers' childrearing time at home but also lowers the opportunity cost of having children. The latter effect tends to raise the number of children. The former increases the wage income of mothers. In this case, parents choose not only to have more children, but also increase educational investment in daughters. It can be the case when the great female labor supply together with a higher female wage rate will increase the total wage income of children's households even with slightly lower educational investment in boys. That is, the quantity-quality tradeoff of children holds only for boys. This case is not apparently inconsistent with the experience in Norway reported by Black et al. (2003).

In contrast, in cases (i) and (iii), a tradeoff exists between quantity and quality for both daughters and sons: Parents prefer quality if the mothers' education level is high, but they prefer quantity with lower mother education level.[25] Presenting the existence of the second case (ii) is a novel result of the present study and a contribution to the literature. Not only are unitary models of family unable to have such a case; even collective models have not shown such a case.

Next, because $w(e_f)/w(e_m) = \ln(1+e_f)/\ln(1+e_m)$, the policy effect on the gender wage ratio is obtained from[26]

$$\text{(28)} \qquad \frac{d}{d\sigma}\left[\frac{w(e_f)}{w(e_m)}\right] = \left[\frac{de_f/d\sigma}{1+e_f} - \frac{w(e_f)}{w(e_m)}\frac{de_m/d\sigma}{1+e_m}\right]/\ln(1+e_m).$$

The gender wage ratio will be greater with the policy in case (ii) and (iii) of Proposition, although the effect is ambiguous in cases (i). In case (i), the change in the wage ratio

25 If $2\varepsilon - \theta\gamma < 0$, then expanded coverage of formal child-care always increases educational investment in daughters, i.e., $de_f/d\sigma < 0$.

26 See Appendix A.2.

depends on the relative magnitudes of the changes in education levels of daughters and sons. The effective female and male wage ratio is definable as the ratio of wage income in a (working) period, $(1-\sigma z n_t) w(e_f)$ and $w(e_m)$. In the literature on efficient bargaining, the representative bargaining powers of a couple are apparently often given by the wage earnings of the spouses rather than their respective wage rates. Therefore, we define here the gender wage earnings gap in terms of this effective wage income ratio as $(1-\sigma z n_t) w(e_f) / w(e_m)$.[27] The policy effect on the gender wage earnings gap is obtained from

(29) $$\frac{d}{d\sigma}[\frac{(1-\sigma z n) w(e_f)}{w(e_m)}] = -zn(1+\frac{\sigma}{n}\frac{dn}{d\sigma})[\frac{w(e_f)}{w(e_m)}] + (1-\sigma z n)\frac{d}{d\sigma}[\frac{w(e_f)}{w(e_m)}].$$

We can show that the effect of the policy change on the gender wage earnings gap is negative, i.e., $d[\frac{(1-\sigma z n) w(e_f)}{w(e_m)}]/d\sigma < 0$, in case (ii) and (iii), although the effects are ambiguous in case (i) of Proposition 1. In case (ii) and (iii), the policy of an expansion of the coverage of formal child-care services reduces the gender wage earnings gap. The policy effect on the gender wage earnings gap depends on the relative magnitudes of the effects on the female and male wage rate and the number of children in other cases.

Finally, we examine the effect of the formal child-care expansion policy on the marriage decisions of women and men. In the steady state, the condition for getting married for women and men, $V^m(e_m, e_f) > w(e_m) - T/2$ and $V^f(e_f, e_m) > w(e_f) - T/2$ can be reduced to[28]

(30) $$\gamma w(e_m) + \gamma(1-zn) w(e_f) + 2u(n) > (1-\gamma)\sigma z n w(e_f) + \frac{p(e_m + e_f)n}{2},$$

Therefore, *ceteris paribus*, the greater the coverage of formal child care becomes (i.e., the smaller σ becomes), the smaller the cost of having children, and the more likely women and men choose to get married rather than to remain single. Expansion of the coverage of formal child care will encourage both women and men to get married and to have children by lowering the costs associated with having children.

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[27] Lundberg et al. (2016) suggested that the gender earnings ratio is a more appropriate measure of the gender wage gap than the wage ratio: w_t^f / w_t^m.

[28] For derivation of (30), see Appendix A.3.

3.2. Numerical example

In order to elucidate the transition to the new steady state after the policy change, we show a simple numerical example. The parameters are set as follows: The utility weight on the welfare of children is $\gamma = 0.45$, which is the same as that for the daughters' welfare reported in Doepke and Tertilt (2009); the utility weight on the number of children is $\varepsilon = 0.66$, which is the same as that in Doepke and Tertilt (2009). Following de la Croix and Doepke (2003), the child-rearing time per child is assumed constant as $z = 0.075$. Assuming that the price of education in terms of goods is $p = 1$, the parameter of the wage function is set at $\theta = 3.8$ so that the number of children of a couple is about 2.1 in the initial steady state.[29]

First, we assume that the fraction of home child rearing is given initially by $\sigma = 0.5$. With the parameters, the initial steady state is characterized by the values $(n, e_f, e_m) = (2.089, 0.5092, 0.6375)$. The wage rates of women and men are $(w(e_f), w(e_m)) = (1.5641, 1.8740)$. Therefore, the gender wage earnings gap is $(1 - \sigma z n) w(e_f) / w(e_m) = 0.7693$. Although the gender wage earnings gap is apparently somewhat higher than the actual gender wage gap prevailing in developed countries, our purpose is to show the effects brought about by the policy change.

Next, we assume that the fraction of at-home child-rearing time is reduced from $\sigma = 0.5$ to $\sigma = 0.3$ in a period, say, in period 3.[30] The new steady state is characterized by $(n, e_f, e_m) = (2.098, 0.5530, 0.6299)$, i.e., the long-term fertility rate and education level of daughters rise but the education level of sons declines.[31] Therefore, this numerical case corresponds to case (ii) in Proposition. The wage rate of

[29] In the present study, following Echevarria and Merlo (1999), we assume that education is a "good" and that its price is constant: unity. de la Croix and Donckt (2010) also assumed that the price of education is equal to that of consumption goods, i.e., $p = 1$ in our notation. We can show that, other thing equal, for a higher price of education, the number of children is greater and the education level of daughters is higher in the steady state in this numerical example (though not shown here).

[30] Because the wage rates of parents are already determined by education investments in the previous period, the decrease in σ shifts line L to the upper-right in Figure 1. The fertility rate and educational investment jump to a point on the shifted line.

[31] Different from us, Iyigun and Walsh (2007b) asserted that asymmetries in the sex ratio in the marriage market can generate the difference in premarital investment between sexes in explanation of a rapid increase in the education of women in the US.

women increases more than the male wage rate does, so that the gender wage ratio $w(e_f)/w(e_m)$ increases from 83.46% in the initial steady state to 90.10% in the new steady state. In this case, not only are the increased education costs because of the increased number of children paid more by fathers than mothers, even though mothers' wage rate rises. The gender wage earnings gap is narrowed to $(1-\sigma z n_t)w(e_{ft})/w(e_m)=0.8585$, although the number of children increases. Consequently, consumption of mothers increases but consumption of fathers decreases, i.e., from $(c_f, c_m)=(0.8186, 1.1285)$ in the initial steady-state changes to $(c_f, c_m)=(0.9061, 1.0899)$ after the policy change. The increased provision of child-care services increases the taxes on couples' total wage income and lowers the opportunity cost of child-rearing of mothers, raising the labor force participation of mothers and therefore their wage income. The raised labor force participation of women induces parents to increase educational investment in daughters. Indeed, child-rearing time at home decreases from 0.0783375 when $\sigma=0.5$ to 0.0472050 when $\sigma=0.3$, although the mothers' working period increases correspondingly.

The transition to the new steady state is depicted in Figure 2. Educational investment in daughters slightly overshoots the long-term equilibrium: parents first increase educational investment in daughters and then increase the number of children moderately, reducing educational investment in sons. As a consequence, the gender wage earnings gap first jumps up and then decreases slightly, remaining stable. In each steady state and during transition, all women and men get married in this numerical example because the utility of each spouse is higher than the threat-point utility.[32]

For other combinations of parameters (p, θ), we can obtain other plausible steady states corresponding to other cases in Proposition. However, what to be noted in this section is that there can be a case in which not only the fertility rate but also the education level of daughters rises when the provision of child-care services is expanded. The implication of the result is that providing child-care services to reduce mothers'

[32] We obtain $V^f-[w(e_f)-T/2]=V^m-[w(e_m)-T/2]=0.594$ when $\sigma=0.5$ and $V^f-[w(e_m)-T/2]=V^m-[w(e_f)-T/2]=0.610$ when $\sigma=0.3$. Therefore, the unmarried threat is not credible in the present case.

child-rearing time at home can enable the economy to have a higher fertility rate and a smaller gender wage earnings gap.

4. Concluding Remarks

We have examined the policy effects of expanding public child care on fertility and educational investment in daughters and sons in a Nash family bargaining model. When the education level of mothers is sufficiently low, the policy will be successful in raising the fertility rate although educational investment in children decreases. If the mothers' education level is sufficiently high, then the policy of expanding the coverage of formal child care lowers the fertility rate but it raises the education levels of children.

A notable result is that a case can exist in which the policy can raise both the education level of daughters and the fertility rate, while lowering the education level of sons. Only in this case can per-capita income and fertility be correlated positively. Recently, in contrast to the result of so-called inverse-J relation between fertility and the Human Development Index (HDI) obtained in Myrskylä et al. (2009), Furuoka (2009) found that higher levels of HDI still tend to be associated with lower fertility rates in countries with high HDI levels. Moreover, Harttgen and Vollmer (2012) asserted that the relation between HDI and the total fertility rate is not robust to United Nations Development Program's recent revision in the HDI calculation method. Furthermore, Luci-Greulich and Thévenon (2014) concluded that fertility increases can be small if economic development is not accompanied by family friendly policies. Our results might shed some light on this argument. We have demonstrated that an expansion of child-care provision can raise both the fertility rate and per-capita income only when the mothers' wage rate is moderate. Results show that the child-care policy rather lowers the fertility rate when the mothers' wage rate is high, although the policy raises wage income per capita.

Up to this point, we have assumed that the utility of a parent depends on the attainable utility of his immediate descendants as well as on the parent's own consumption. Whether such altruistic behaviors of parents actually hold or not has been argued (e.g., Hayashi, 1995). The matter remains unsettled. In interpreting the result in the numerical example in relation to the reality, one must be sufficiently attentive.

Second, bargaining between women and men has been assumed to occur only once in the present study. However, bargaining within a couple can be multi-stages. Iyigun and Walsh (2007a), among others, analyzed such a bargaining model. The analysis might be extended to multi-period-lifetime settings. This is a promising direction of future research.

Finally, we have assumed the wage function of the Mincer type. Doepke and Tertilt (2009) assumed human capital accumulation with intergenerational externalities and showed the time paths along the accumulation path of human capital stock. Considering human capital accumulation from generation to generation is expected to be an important expansion on the analysis of family policy in the future.

Appendix

A.1 Uniqueness and Stability of Steady States

The steady-state values of the number of children and educational investment in daughters, n and e_f, are given respectively by the following equations:

$$n = \frac{2}{\frac{1}{\gamma\theta}[\frac{2\varepsilon}{n} - \sigma z\theta \ln(1+e_f) + p] + \sigma z}, \quad \text{(A1)}$$

$$1 + e_f = \frac{1}{p}[\frac{2\varepsilon}{n} - \sigma z\theta \ln(1+e_f) + p] - \frac{\gamma\theta\sigma z}{p}. \quad \text{(A2)}$$

In order to examine local uniqueness and stability of the steady state, linearizing the dynamic system (17) and (18) around the steady state, we obtain the Jacobian matrix as

$$J(n, e_f) = \begin{vmatrix} \frac{\varepsilon}{\gamma\theta} & \frac{\sigma z n^2}{2\gamma(1+e_f)} \\ -\frac{2\varepsilon}{pn^2} & -\frac{\sigma z\theta}{p(1+e_f)} \end{vmatrix}. \quad \text{(A3)}$$

Denoting the determinant and the trace of matrix $J(n, e_f)$ by $D(J)$ and $T(J)$, we obtain

$$D(J) = -\frac{\varepsilon}{\gamma\theta}\frac{\sigma z\theta}{p(1+e_f)} + \frac{2\varepsilon}{pn^2}\frac{\sigma z n^2}{2\gamma(1+e_f)} = 0, \quad \text{(A4)}$$

$$T(J) = \frac{\varepsilon}{\gamma\theta} - \frac{\sigma z\theta}{p(1+e_f)} = \frac{\varepsilon p(1+e_f) - \gamma\sigma z\theta^2}{\gamma\theta p(1+e_f)}. \quad \text{(A5)}$$

We have three cases: (i) If both $|1+D(J)|>|T(J)|$ and $|D(J)|<1$ hold, then both eigenvalues are inside the unit interval and the two associated manifolds are stable around the steady state. (ii) If $|1+D(J)|>|T(J)|$ and $|D(J)|>1$ hold, then both eigenvalues are outside of the unit interval and the associated manifolds are unstable around the steady state. (iii) If $|1+D(J)|<|T(J)|$, then one eigenvalue is inside of the unit interval and the other is outside of the unit interval. One stable and one unstable manifold exist around the steady state. The steady state is (saddle-point) unstable in this case.

In the present dynamic system of (17) and (18), because we have $|D(J)|=0$ and $1>T(J)$ when $\varepsilon-\theta\gamma<0$, only case (i) can be possible.

Next, from (17) and (18), one obtains

(A6) $$n_{t+1}=\frac{1}{\frac{p}{2\gamma\theta}(1+e_{ft+1})+\sigma z}.$$

The fertility rate and mother's education level in each period must satisfy condition (A6), which is illustrated as a dotted line L in Figure 1. Letting $(n^*,e_f{}^*)$ be the stable steady state, the dynamics can be illustrated by arrows in Figure 1. Assuming that the initial condition is given as (n_{t_0-1},e_{ft_0}) in period t_0 and the initial point is not on line L, the fertility rate in period t_0 jumps to a point corresponding to e_{ft_0} on line L, and then the system converges to the steady state S along the line. The steady state is unique and stable, if it exists.

A.2 Policy effects on gender wage and earnings gaps

From (28), in case (ii), because $de_f/d\sigma<0$ and $de_m/d\sigma>0$, we can readily show that the right hand side of (28) is negative. In case (iii), from (14) and (15), one can obtain

(A7) $$\frac{de_f}{d\sigma}=-\frac{2\gamma\theta}{p}(\frac{1}{n^2}\frac{dn}{d\sigma}+z)<\frac{de_m}{d\sigma}=-\frac{2\gamma\theta}{p}\frac{1}{n^2}\frac{dn}{d\sigma}\leq 0$$

and, because $e_m > e_f$, $\frac{1}{1+e_f} - \frac{w(e_f)}{w(e_m)} \frac{1}{1+e_m} > 0$. Therefore, the right-hand side of (28) is negative in case (iii).

In case (ii), the second term on the right-hand side of (29) is negative. From (A7), $de_f / d\sigma < 0$ means that $0 > dn/d\sigma > -n^2 z$, which can be rewritten as $1+(\sigma/n)(dn/d\sigma) > 1-\sigma zn > 0$. Therefore, the right-hand side of (29) is negative in case (ii).

A.3 Condition for Marriages

In the steady state, we have

(A8) $$V^m(e_m,e_f) = c_m + u(n) + \frac{\gamma}{2}[V^m(e_m,e_f) + V^f(e_f,e_m)]$$

$$= w(e_m) - \frac{znw(e_f) + \frac{p(e_m+e_f)}{2}n}{2} + u(n)$$

$$+ \frac{\gamma}{2}[V^m(e_m,e_f) + V^f(e_f,e_m)],$$

(A9) $$V^f(e_f,e_m) = c_f + u(n) + \frac{\gamma}{2}[V^m(e_m,e_f) + V^f(e_f,e_m)]$$

$$= w(e_f) - \frac{znw(e_f) + \frac{p(e_m+e_f)}{2}n}{2} + u(n)$$

$$+ \frac{\gamma}{2}[V^m(e_m,e_f) + V^f(e_f,e_m)].$$

where we use (12) and (13). From (A8) and (A9), we obtain

(A10) $$V^m(e_m,e_f) = w(e_m) - \frac{znw(e_f) + \frac{p(e_m+e_f)}{2}n}{2} + u(n)$$

$$+ \frac{1}{2}\sum_{t=1}^{\infty}\gamma^t [w(e_m) + \{1-[\sigma+\delta(1-\sigma)]zn\}w(e_f) - \frac{p(e_m+e_f)n}{2} + 2u(n)]$$

$$= w(e_m) - \frac{znw(e_f) + \frac{p(e_m + e_f)}{2}n}{2} + u(n)$$

$$+ \frac{\gamma}{2(1-\gamma)}[w(e_m) + \{1 - [\sigma + \delta(1-\sigma)]zn\}w(e_f) - \frac{p(e_m + e_f)n}{2} + 2u(n)].$$

For men to get married, both conditions $V^m(e_m, e_f) > w(e_m) - T/2$ and $\delta = 1$ must hold, where $T = (1-\sigma)znw(e_f)$. From (A10), it follows that

(30) $$\gamma w(e_m) + \gamma(1 - zn)w(e_f) + 2u(n) > (1-\gamma)\sigma znw(e_f) + \frac{p(e_m + e_f)n}{2},$$

where $(1 - zn)w(e_f) = (1 - \sigma zn)w(e_f) - T$. The left-hand side of (30) is the utility obtained from having children. The right-hand side represents the cost of having children. Similarly, it can be shown that condition $V^f(e_f, e_m) > w(e_f) - T/2$ for women is also reduced to (30).

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Figure 1 Dynamics of the system

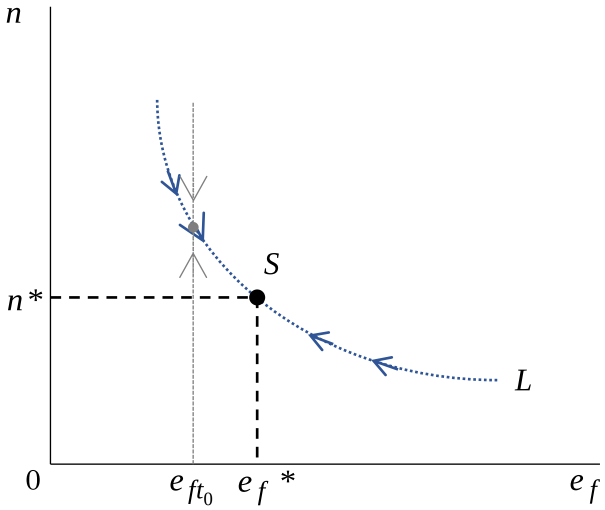
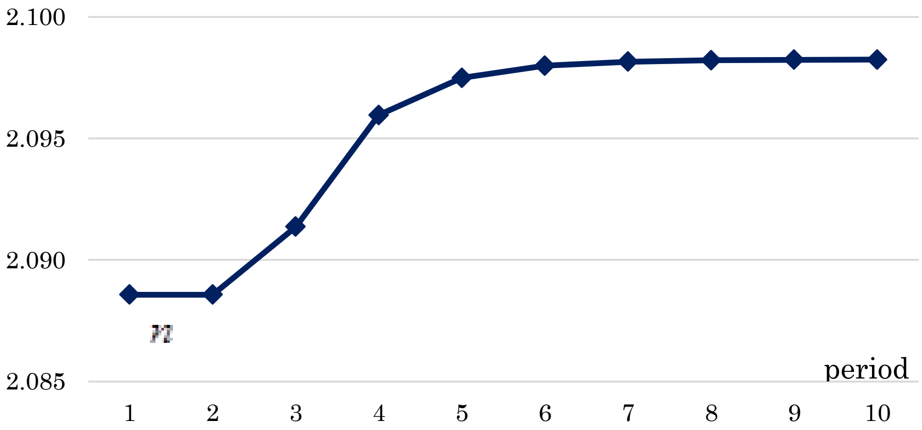
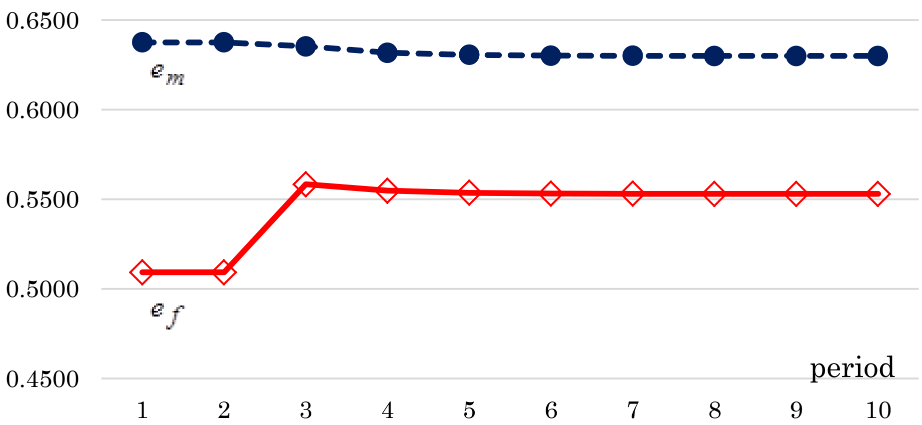


Figure 2 Effects of expansion of child-care provision ($\sigma = 0.5$ to $\sigma = 0.3$ in period 3)

(a) Number of children



(b) Education levels



(c) Gender wage earning gap