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An Endogenous Objective Function of a Partially Privatized Firm: A Nash Bargaining Approach*

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Abstract

We establish a model where a partially privatized firm's objective function is endogenously determined by considering Nash bargaining between the owners.

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1 Introduction

Partial privatization means that privatized firms are owned by both private and public sectors. If firms are owned by one type of owners, the managers should operate their firms to maximize the owners' objectives. For example, as is usually seen in oligopoly theory, the managers of private firms set outputs to maximize profits. However, when we consider firms owned by two (or more) types of owners, what

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do the managers maximize? This article analyzes the endogenous determination of such firms' objective functions.

Matsumura (1998) first formulated the objective functions of the partially privatized firm as the weighted average of owners' objective function, that is, $V = \alpha W + (1-\alpha)\Pi$, where W is welfare, which is the government's objective, and Π is the profit, which is the private shareholders' objective.[1] He assumed that the share held by the government, s, is positively related to the weight, α. In other words, progress in privatization makes the manager put more emphasis on profits in the partially privatized firm's objective function. The problem arises from the fact that he presumed the positive correlation between α and s a priori without any theoretical foundation.

We explore the relationship of α and s by using Nash bargaining over α between owners. In this model, the manager sets the output of the partially privatized firm to maximize V, which has a weight on welfare α determined through bargaining for a given share s.

2 Model and Results

We consider an industry where a partially privatized firm (firm 0) and a private firm (firm 1) are engaged in Cournot competition. These firms produce a homogeneous commodity, and demand for this commodity is represented by the inverse demand function $P = P(Q) = 1-Q$. Here, P represents the price; $Q = q_0 + q_1$, the total quantity produced by the two firms; and q_i, the output of the firm i ($i = 0, 1$). Let the cost functions of these firms be given as $C_i(q_i) = c_i q_i$, and we assume that $c_0 = c > 0 = c_1$.[2] Firm i's profit is

$$\Pi_i(q_0, q_1) = (P(Q) - C_i(q_i))\, q_i,$$

and social welfare is

$$W(q_0, q_1) = \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1).$$

Firm 1, which is owned by only dividend-maximizing private shareholders, is assumed to be a

[1] Matsumura's (1998) formulation is prevalent in the mixed oligopoly theory whose pioneering work is by De Fraja and Delbono (1989). For recent studies based on Matsumura's, see Mukherjee and Suetrong (2009) and Heywood and Ye (2010).

[2] This assumption of the partially privatized firm's inefficiency is standard in a mixed oligopoly with linear demand and costs. It guarantees that all the firms are active in the market. In addition, empirical studies substantiate the inefficiency of partially privatized firms and fully nationalized firms. For examples, see Megginson and Netter (2001) and Fries and Taci (2005).

profit-maximizer. On the other hand, firm 0 is owned by the dividend-maximizing private shareholders and the welfare-maximizing government. Then, following Matsumura (1998), we assume that firm 0 (or the manager of firm 0) maximizes the weighted average of social welfare and its profit, given as

$$V_0(q_0, q_1, \alpha) = \alpha W(q_0, q_1) + (1-\alpha)\Pi_0(q_0, q_1), \quad \text{where} \quad \alpha \in [0,1].$$

The parameter α is a degree of how the manager should reflect the government's objective in firm 0's objective function.

We assume that the government owns a share of $s \in (0,1)$ in the partially privatized firm and the private shareholders own a share of $1-s$. In this case, a decrease in s indicates further progress in privatization. In proportion to their shares, the two types of owners receive their dividends from the firm's profit: $s\Pi_0$ and $(1-s)\Pi_0$ for the government and the private shareholders, respectively. Given their shares, these owners bargain over α in the objective function of the privatized firm. Matsumura (1998) assumed that α is an increasing function of s, but in our paper, α is independent from s a priori.

The game in this paper runs as follows.

Stage 1: The two parties engage in Nash bargaining over weight $\alpha \in [0,1]$. If they reach an agreement on the value of α, stage 2 follows; otherwise, they receive the disagreement point payoff $d = (d_g(s), d_p(s))$.

Stage 2: The partially privatized firm, with the agreed weight α in stage 1, and the private firm compete in Cournot fashion.

At stage 2, the first-order conditions, $\partial V/\partial q_0 = 0$ and $\partial \Pi_1/\partial q_1 = 0$, yield the following equilibrium outcomes:

$$q_0^*(\alpha) = \frac{1-2c}{3-2\alpha}, \quad q_1^*(\alpha) = \frac{1-\alpha+c}{3-2\alpha}, \quad Q^*(\alpha) = \frac{2-\alpha-c}{3-2\alpha}, \tag{1}$$

$$\Pi_0^*(\alpha) = \frac{(1-\alpha)(1-2c)^2}{(3-2\alpha)^2}, \quad \Pi_1^*(\alpha) = \frac{(1-\alpha+c)^2}{(3-2\alpha)^2}, \quad \text{and} \tag{2}$$

$$W^*(\alpha) = \frac{(11-8\alpha)c^2 - 2(4-3\alpha)c + 8 - 10\alpha + 3\alpha^2}{2(3-2\alpha)^2}. \tag{3}$$

For the subsequent analysis, we make the following assumption:

Assumption 1. *The partially privatized firm's marginal cost is sufficiently low, that is, $c < 1/6$.*

Let $\alpha_g = \text{argmax}_\alpha W^*(\alpha)$ and $\alpha_p = \text{argmax}_\alpha \Pi_1^*(\alpha)$. It is easily calculated that $\alpha_p = 1/2$ and $\alpha_g = (1-5c)/(1-4c)$. Under Assumption 1, $\alpha_g > \alpha_p$ holds and, moreover, we have $W^{*\prime}(\alpha) > 0$ and $\Pi^{*\prime}(\alpha) < 0$ for any $\alpha \in (\alpha_p, \alpha_g)$. The relationship reveals that in the interval $[\alpha_p, \alpha_g]$, the owners' interests conflict; therefore, they have to agree on some values of α through bargaining and decide on α in the interval.

We now proceed to stage 1. At this stage, the government's payoff is $U_g(\alpha) = W^*(\alpha)$, whereas that of the private shareholders is $U_p(\alpha, s) = (1-s)\Pi_0^*(\alpha)$. With the basic assumption of the *free disposal of utility*, the feasible set of payoffs through bargaining is defined as $A = \{(u_g, u_p) \in \mathbb{R}^2 \ : \exists \alpha \in [\alpha_p, \alpha_g]$ such that $U_g(\alpha) \geq u_g$ and $U_p(\alpha, s) \geq u_p\}$. As for this feasible set, we have the following lemma.

Lemma 1. *The feasible set of our bargaining problem, A, is a convex set under Assumption 1.*

Proof. See Kamijo and Tomaru (2008). □

Let a pair (A, d) represent a *bargaining problem for the partially privatized firm's objective*. In order to make this bargaining problem plausible, we assume the following:

Assumption 2. *The disagreement point $(d_g(s), d_p(s))$ is in the interior of A.*

Lemma 1 and Assumption 2 ensure the existence and uniqueness of the Nash solution. The Nash solution (U_g^*, U_p^*) is simply connected to the agreed value of α. Let α^* denote the solution to the following maximization problem:

$$\max \ (U_g(\alpha) - d_g)(U_p(\alpha, s) - d_p) \ \text{ s.t. } \alpha \in [\alpha_p, \alpha_g]. \tag{4}$$

In our setting, maximization problem (4) has an interior solution.[3] Thus, $\alpha^* \in (\alpha_p, \alpha_g)$. Since the solution of the maximization problem depends on the share s, we write $\alpha^*(s)$ instead of α^*.

We obtain the following proposition on the comparative statics of α^* with respect to s.

Proposition 1. *For $s \in (0,1)$, the sign of $\alpha^{*\prime}(s)$ coincides with the sign of the following equation:*

$$-\frac{\partial U_p}{\partial \alpha} d_g'(s) - U_g'(\alpha^*)\left(d_p'(s) + \frac{d_p(s)}{1-s}\right).$$

[3] For concrete proof of this, see Lemma 1's proof in Kamijo and Tomaru (2008).

Proof. Thus, the first-order condition yields $U_g'(\alpha^*(s))(U_p(\alpha^*(s), s)-d_p(s))+(\partial U_p/\partial\alpha)(U_g(\alpha^*(s))-d_g(s)) = 0$. Let $H(\alpha, s) = (U_g(\alpha) - d_g(s))(U_p(\alpha, s) - d_p(s))$. From the implicit function theorem, the sign of $\alpha^{*\prime}(s)$ is the same as that of $\partial^2 H/\partial s\partial\alpha$. Noting that $\partial U_p/\partial s = -U_p(\alpha, s)/(1 - s)$ and $\partial^2 U_p/\partial s\partial\alpha = -(\partial U_p/\partial\alpha)/(1 - s)$, we have

$$\begin{aligned}\left.\frac{\partial^2 H}{\partial s\partial\alpha}\right|_{\alpha=\alpha^*(s)} &= U_g'(\alpha^*)\left(\frac{\partial U_p}{\partial s} - d_p'(s)\right) + \frac{\partial^2 U_p}{\partial s\partial\alpha}\left(U_g(\alpha^*) - d_g(s)\right) - \frac{\partial U_p}{\partial\alpha}d_g'(s),\\ &= U_g'(\alpha^*)\left(\frac{U_p}{1-s} - d_p'(s)\right) - \frac{1}{1-s}\frac{\partial U_p}{\partial\alpha}\left(U_g(\alpha^*) - d_g(s)\right) - \frac{\partial U_p}{\partial\alpha}d_g'(s).\end{aligned}$$

Using the first order condition on the above equation, we obtain the desired results. □

3 Applications

Proposition 1 says that the comparative statics of α^* with respect to s is determined by how the disagreement point (d_g, d_p) depends on the share s of the public sector in the partially privatized firm. The relationship between the disagreement point and s is determined by what happens after the negotiation breaks down. We consider two different scenarios after the breakdown.

1. Resorting to voting powers at a shareholders' meeting. Suppose that the majority party resorts to its voting powers at a shareholders' meeting after the bargaining breakdown. In this senario, their payoffs $e_g(s)$ and $e_p(s)$ after the negotiation are as follows: when $s < \frac{1}{2}$ (i.e., when the private sector is in the majority), $e_g(s) = U_g(0)$ and $e_p(s) = U_p(0, s)$, and when $s > \frac{1}{2}$ (i.e., when the public sector is in the majority), $e_g(s) = U_g(1)$ and $e_p(s) = U_p(1, s)$.

Proposition 2. *Assume* $(d_g, d_p) = (e_g, e_p)$*. In that case,* $\alpha^{*\prime}(s) = 0$ *for any* $s \in (0, 1/2) \cup (1/2, 1)$.

Proof. For the disagreement of this proposition, $d_g'(s) = 0$ and $d_p'(s) + \dfrac{d_p(s)}{1-s} = -\Pi_0^*(\alpha^*(s)) + \Pi^*(\alpha^*(s)) = 0$. Applying these two to the Proposition 1's condition, we have $\alpha^{*'}(s) = 0$. □

Proposition 2 states that privatization (i.e., a decrease in s) does not influence α if the major shareholder resorts to his voting power at the shareholders' meeting after the bargaining breakdown.

When $s = 0.5$, the voting powers of the two parties at a shareholders' meeting are even. In this case, the second scenario can be appropriate.

2. Defunding the partially privatized firm. Suppose that after the negotiation breakdown, the government and the private shareholders defund and liquidate the partially privatized firm. As a result, the money invested is returned to both owners. Subsequently, the owners invest the refunded money in other investment avenues. Let K, r_p, and r_g denote the total amount of investment in the firm, the return rate on other investments of private shareholders, and that of the government, respectively. Thus, the private shareholder obtains

$$b_p = r_p(1-s)K.$$

Since the firm is liquidated, the remaining private firm 1 monopolizes the market. Therefore, social welfare after the negotiation breakdown is the sum of the welfare in private monopoly and the returns from the investments of both parties. The government's payoff b_g is

$$b_g = W_M + r_g sK + r_p(1-s)K = \frac{3}{8} + \left[r_g s + r_p(1-s)\right] K,$$

where $W_M = 3/8$ represents welfare in private monopoly.

Proposition 3. *Let* $s \in (0,1)$. *Assuming that* $(d_g, d_p) = (b_g, b_p)$,

$$r_p \gtreqless r_g \quad \Longleftrightarrow \quad \alpha^{*\prime}(s) \gtreqless 0.$$

Proof. For the disagreement point $(d_g, d_p) = (b_g, b_p)$, it is easily calculated that

$$-\frac{\partial U_p}{\partial \alpha} d_g'(s) - U_g'(\alpha^*) \left(d_p'(s) + \frac{d_p(s)}{1-s} \right) = \frac{\partial U_p}{\partial \alpha} K(r_g - r_p).$$

Since $\frac{\partial U_p}{\partial \alpha} = (1-s)\Pi_0^{*\prime}(\alpha) < 0$ for $\alpha \in (\alpha_p, \alpha_g)$, the desired results are obtained from Proposition 1. □

Proposition 3 states that privatization could either increase or decrease α if the privatized firm is liquidated and the money invested is returned to owners after the bargaining breakdown.

4 Concluding remarks

We considered the endogeneous determination of the partially privatized firm's objective function. Matsumura (1998) assumed that privatization monotonically decreases the weight on welfare in the partially privatized firm's objective function. This paper presented the possibilities of collapsing his assumption by considering Nash bargaining between the owners of the partially privatized firm. Nevertheless, when the return rate of private investment r_p is higher than that of public investment r_g, which may be a natural economic environment, Matsumura's (1998) assumption is correct.

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